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Optimal design in a problem of spatial ecology

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Abstract

The paper is mainly concerned with the application of optimal design concepts in the area of biodiversity. Statistical techniques for detecting spatial patterns in the distribution of species richness now have some long tradition in this field, specifically the use of correlograms. The issue of where (and when) to undertake observations has but only rarely been treated. In this paper we aim to extend the existing literature with techniques on finding good designs to optimize the power of tests for spatial dependence. Special emphasis will be given to the difference in using the exact distribution of Moran's $\mathcal{I}$ and its normal approximation in this context. Two illustrative artificial examples will be followed by a real case analysis from the ecological literature.
Keywords: Biodiversity,Species Richness , Spatial Sampling

1 Introduction

In population ecology there exist considerable interest in precise estimation of the amount of spatial autocorrelation of species richness, see Field et al (2009) for a recent survey and taxonomy of studies. In these studies the role and impact of spatial autocorrelation has increasingly been acknowledged, ever since the influential paper of Legendre (1993) has paved the way. It has been realized that it may strongly affect statistical analysis by potentially inflating Type I errors and reducing power, for a recent discussion of these issues see Dale and Fortin (2009). Moreover, the "red herrings" debate (cf. Lennon (2000) and Diniz-Filho et al (2003)) demonstrates that researches in ecology have clearly become aware of what is at stake, when such phenomena are neglected.

Plotting measures of spatial autocorrelation against observation distances, the so-called correlogram has become the main tool for assessing spatial dependence in species richness (see eg. Fortin and Dale (2005)). Those correlograms are predominantly based upon the Moran's $\mathcal{I}$, a measure introduced by Moran (1950) for the use in geological applications, and we will thus concentrate our investigations on it throughout the paper. However, note that the principal considerations we propose are not limited to this choice and any other of the available measures (see e.g. Dale et al (2002) for a survey) could be equally employed.

It has been noticed comparatively early, see Fortin et al (1989) and Dutilleul (1993), that the locations of where observations are to be taken, the sampling design, has an important impact on the ability to detect spatial autocorrelation and thereby on the validity of statistical conclusions drawn from the gathered data. Although considerable efforts have been undertaken to deduct from simulation studies, which sampling designs should be preferred (see Legendre et al (2002) as a particularly informative paper), rarely have there been suggestions on how to actively construct optimal designs for estimating biodiversity (a recent notable exception being Archaux and Bergès (2008), who, however, do not take into account the role of spatial autocorrelation).

This is exactly what was intended that this paper shall provide. Particularly, we employ and extend a procedure that maximizes the power of detecting spatial autocorrelation measured by the Moran's $\mathcal{I}$ in spatial species richness studies. Therewith we intend to provide a blueprint for statistical sampling optimizations in general biodiversity research. We provide a couple of constructed generic examples as well as an application to a real case. As a side issue we also investigate, whether the use of the common normal approximation of the distribution of Moran's $\mathcal{I}$ has any impact on the quality of the sampling design.

Throughout the paper we will assume the data to be generated by Gaussian simultaneous spatial autoregressive (SAR) process, i.e.

$$y = \beta + \rho V y + \varepsilon \qquad \text{with} \qquad \varepsilon \sim N(0, \sigma^2 \cdot I_n) \tag{1}$$

where $y^T = (y_1, \ldots, y_n)$ and V is the spatial link matrix, which yields an observed covariance of $\sigma^2 \Omega$, with

$$\Omega^{\frac{1}{2}} = \left(I - \rho V^T\right)^{-1}$$

This is equivalent to

$$y = \beta' + \eta \qquad \text{with} \qquad \eta \sim N(0, \sigma^2 \cdot \Omega) \tag{2}$$

$$\text{where} \quad \eta = \rho V \eta + \varepsilon \quad \text{and} \quad \beta' = \left(I - \rho V^T\right)^{-1} \beta$$

We can consider $\beta = 0$ without loss of generality, and we can also easily add regressors to model (1) if necessary.

2 Distribution of Moran's I

For the use in regression analysis Moran's $\mathcal{I}$ is defined as scale invariant ratio of quadratic forms in the normally distributed regression residuals $\hat{\eta} = (\hat{\eta}_1, ..., \hat{\eta}_n)^T$, i.e.

$$\mathcal{I} = \frac{\hat{\eta}^T \frac{1}{2}(V + V^T)\hat{\eta}}{\hat{\eta}^T \hat{\eta}} \tag{3}$$

where $\sum_{i=1}^{n}\sum_{i'=1}^{n} v_{ii'} = n$, see e.g. Fortin and Dale (2005).

The Moran's $\mathcal{I}$ test is used for parametric hypotheses about the spatial autocorrelation level ρ, i.e. $H_0 : \rho = 0$ against $H_A : \rho > 0$ for positive spatial autocorrelation; or $H_0 : \rho = 0$ against $H_A : \rho < 0$ for negative spatial autocorrelation. The former tests are much more relevant in practice, because negative spatial autocorrelation very rarely appears in species richness data. Thus, from now on $\rho \geq 0$ will be assumed.

2.1 The normal approximation

One of the most commonly used methods is to employ its asymptotic normal distribution (derived by Cliff and Ord (1973)) as an approximation, and test its standardized value

$$z(\mathcal{I}) = \frac{\mathcal{I} - E[\mathcal{I}|H_0]}{\sqrt{V[\mathcal{I}|H_0]}} \sim N(0,1). \tag{4}$$

against a standard normal.

For this purpose and the purpose of power evaluation one thus requires to evaluate the moments under the assumption of spatial independence as well as under a given spatial process for the alternative. Here, we will just give brief representations, for a more detailed treatment see Tiefelsdorf (2000).

For a random variable Y, measured in each of the n non-overlapping subareas of the whole study area, Moran's $\mathcal{I}$ is defined from the residuals of an intercept only regression, i.e. $\hat{\eta} = My$ where $M = I_n - \frac{1}{n}1_n1_n^T$, I_n is an $n \times n$ identity matrix and 1_n is an $n \times 1$ vector of ones. In this case, and if the spatial link matrix V has full rank, i.e. there is no observation completely separated from all others, the expected value of the test statistic $\mathcal{I}$ under the assumption of spatial independence is given by

$$E\left[\mathcal{I}\,|H_0\right] = -\frac{1}{n-1} = \bar{\gamma} \tag{5}$$

and its variance is

$$V\left[\mathcal{I}\,|H_0\right] = \frac{2n}{n^2-1}\sum_{i=1}^{n}(\gamma_i - \bar{\gamma})^2 = \frac{2n}{n^2-1}\sigma_\gamma^2 \tag{6}$$

where $\{\gamma_1, ..., \gamma_{n-1}, 0\}$ are the eigenvalues of the matrix $M \cdot \frac{1}{2} \cdot \left(V + V^T\right) \cdot M$. Besides $\mathcal{I}$ is asymptotically normally distributed.

Under the influence of a spatial process the random errors η are normally distributed with covariance matrix $\sigma^2\Omega$, and hence the regression residuals $\hat{\eta}$ are normally distributed with covariance matrix $\sigma^2 M\Omega M$. The expectation of the random errors η and the expectation of the regression residuals $\hat{\eta}$ are zero, which is eventually important as it leads to central χ^2-distributed variables. The structure of the matrix Ω depends upon the spatial process that is assumed to generate the data under the alternative.

Let us then define

$$H \equiv P^T \cdot \Omega^{T\frac{1}{2}} \cdot M \cdot \frac{1}{2} \cdot \left(V + V^T\right) \cdot M \cdot \Omega^{\frac{1}{2}} \cdot P$$

and

$$\Lambda \equiv P^T \cdot \Omega^{T\frac{1}{2}} \cdot M \cdot \Omega^{\frac{1}{2}} \cdot P$$

where Λ is a $n \times n$ diagonal matrix of eigenvalues λ_i of $\Omega^{T\frac{1}{2}} \cdot M \cdot \Omega^{\frac{1}{2}}$ and P is a $n \times n$ matrix whose columns are the normalized eigenvectors of $\Omega^{T\frac{1}{2}} \cdot M \cdot \Omega^{\frac{1}{2}}$. Because of the rank defect of the projection matrix M only $n-1$ eigenvalues of $\Omega^{T\frac{1}{2}} \cdot M \cdot \Omega^{\frac{1}{2}}$ are non zero.

The conditional expectation and variance of Moran's $\mathcal{I}$ in this case of spatial dependence can be derived from its two first moments given by

$$E\left[\mathcal{I} \mid H_1\right] = \int_0^\infty \theta(t, \Lambda) \cdot \sum_{i=1}^{n-k} \frac{h_{ii}}{1 + 2 \cdot \lambda_i \cdot t} dt$$

$$E\left[\mathcal{I}^2 \mid H_1\right] = \int_0^\infty \theta(t, \Lambda) \cdot \sum_{i=1}^{n-k} \sum_{j=1}^{n-k} \frac{(h_{ii} \cdot h_{jj} + 2 \cdot h_{ij}^2) \cdot t}{(1 + 2 \cdot \lambda_i \cdot t) \cdot (1 + 2 \cdot \lambda_j \cdot t)} dt,$$

where $\theta(t, \Lambda) = \prod_{i=1}^{n-k} (1 + 2 \cdot \lambda_i \cdot t)^{-\frac{1}{2}}$ and h_{ij} are the elements of the matrix H.

2.2 The exact distribution

The exact small sample distribution of Moran's $\mathcal{I}$ was obtained seemingly independently by Tiefelsdorf and Boots (1995) and Hepple (1998). Bivand et al (2009) have shown that considerable losses in accuracy can occur and that especially for power computations the errors induced by the normal approximation can be severe. Due to the permanently increasing computing power in our days it became possible and worthwhile to evaluate the numerically demanding exact distribution of the Moran's $\mathcal{I}$ statistic in shorter time for even big lattices.

Under the influence of a spatial process the conditional distribution of Moran's $\mathcal{I}$ given the observed value $\mathcal{I}_0$ and a hypothetical spatial process generating $\sigma^2\Omega$ can be written as (see Tiefelsdorf (2000))

$$\begin{aligned} F(\mathcal{I}_0|H_1) &= P\left(\frac{\delta^T \cdot \Omega^{T\frac{1}{2}} \cdot M \cdot \frac{1}{2} \cdot \left(V + V^T\right) \cdot M \cdot \Omega^{\frac{1}{2}} \cdot \delta}{\delta^T \cdot \Omega^{T\frac{1}{2}} \cdot M \cdot \Omega^{\frac{1}{2}} \cdot \delta} \leq \mathcal{I}_0\right) \\ &= P\left(\delta^T \cdot \Omega^{T\frac{1}{2}} \cdot M \cdot \left[\frac{1}{2} \cdot \left(V + V^T\right) - \mathcal{I}_0 \cdot I_n\right] \cdot M \cdot \Omega^{\frac{1}{2}} \cdot \delta \leq 0\right) \end{aligned}$$

where δ is a random variable with distribution $N(0, I_n)$. By the spectral decomposition Theorem, the matrix

$$L_{H1} \equiv \Omega^{T\frac{1}{2}} \cdot M \cdot \left[\frac{1}{2} \cdot \left(V + V^T\right) - \mathcal{I}_0 \cdot I_n\right] \cdot M \cdot \Omega^{\frac{1}{2}} \tag{7}$$

(note that L_{H1} is symmetric) can be written as $L_{H1} = A^T \cdot \Phi \cdot A$, where A is the matrix of the normalized eigenvectors and $\Phi = diag(\phi_1, ...\phi_n)$ is the diagonal eigenvalue matrix of Ł$_{H1}$ given in equation (7). Substituting into equation (7) we get

$$F\left(\mathcal{I}_0\,|H_1\right) \;\; = P\left(\delta^T \cdot A^T \cdot \Phi \cdot A \cdot \delta \leq 0\,|H_1\right)$$

Because the random error vector δ belongs to the class of the spherically symmetric distributions, the orthogonal transformation $\tau \equiv A \cdot \delta$ is again independent normally distributed with $\tau \sim N\left(0, I_n\right)$ (cf. Tiefelsdorf (2000)).

So the conditional distribution of Moran's $\mathcal{I}$ given by

$$F\left(\mathcal{I}_0\,|H_1\right) \;\; = P\left(\sum_{i=1}^n \phi_i \cdot \tau_i^2 \leq 0\,|H_1\right), \tag{8}$$

enables us to use Imhof's formula (see eg. Broda and Paolella (2009)) because $\sum_{i=1}^n \phi_i \cdot \tau_i^2$ is a weighted sum of χ_1^2-distributed variables. The solution of the integral in Imhof's formula can be approximated by numerical integration with the behavior of the improper integral at the boundaries considered with special starting and truncation values (see again Tiefelsdorf (2000) for details).

3 Optimal spatial design

The set of locations of spatial data collection sites (the so-called design) influences decisively the quality of the results of the statistical analysis. Usually in choosing the design the aim is to ensure continuous monitoring of a data generating process or to allow for point prediction of present or future states of nature. Here the primary focus is on the ability of the spatial network to efficiently detect the presence of spatial autocorrelation.

Sampling theory and optimum experimental design theory are two large branches in theoretical statistics that have developed separately, though with considerable theoretical overlap, both of them providing methods for efficient site positioning. Whereas sampling theory is a basically model-free methodology essentially oriented towards restoring unobserved data, in optimum design theory the aim is to estimate the structure of the data generating process e.g. the parameters of an assumed (regression) model or functions of these parameters. There, the basic approach is to define a so-called design criterion, that reflects the overall aim of the data-gathering process and to derive corresponding methods for optimizing this criterion through selecting efficient observation sites.

This setup makes it a particularly suitable framework for our purpose. Thus we will concentrate in the following on methods borrowed from the optimal design area, such as for instance recently reviewed in Müller (2007). Complementary reviews of the model-free approaches to spatial design are e.g. given in de Gruijter et al (2006).

3.1 The power criterion

It is natural that in our attempt to optimize a design to detect spatial autocorrelation we would strive for maximizing the power of a respective test. Gumprecht et al (2009) have introduced the power of Moran's I test as design criterion, albeit with its commonly used normal approximation. The general aim is equivalent to minimizing the probability that, given the alternative, the Moran's $\mathcal{I}$ test accepts the null hypothesis of no spatial autocorrelation, which is yielded from the conditional distribution (8)

$$\min F(k_0|H_1), \text{ or equivalently } \max J_{\mathcal{I}} = 1 - F(k_0|H_1),$$

where k_0 denotes the corresponding critical value. Note that this could be taken as a quantile from either the exact distribution $F(\mathcal{I}_0|H_0)$ or from the normal approximation, for which it would simply yield $k_0 = \Phi^{-1}(1-\alpha)\sqrt{V(\mathcal{I}|H_0)} + E(\mathcal{I}|H_0)$. In case of using the normal approximation twice, the final criterion to be maximized is then explicitely given by (cf. Gumprecht et al (2009))

$$\tilde{J}_{\mathcal{I}} = 1 - \Phi\left(\frac{\Phi^{-1}(1-\alpha)\sqrt{V[\mathcal{I}|H_0]} + E[\mathcal{I}|H_0] - E[\mathcal{I}|H_A]}{\sqrt{V[\mathcal{I}|H_A]}}\right) \tag{9}$$

using (5) and (6) and (8). Note, however, that in our attempt to find exact values, we need embedded evaluations of Imhof's formula as described in Subsection 2.2.

Unfortunately both the given criteria can not expected to be convex and thus we can not employ the powerful machinery from the well developed optimum design theory (cf. eg. Atkinson et al (2007)), but must resort to alternative ad-hoc algorithmic approaches, as given below.

3.2 Algorithms

3.2.1 Full enumeration

Evidently, the global optimal design can be found by evaluating all possible designs, i.e. in an m-point grid there are $\binom{m}{r}$ possible r-point designs, r here goes from 4 to m. The number of possible designs increases very fast with the size of the grid leading to a high runtimes induced by the Imhof formula, as the numerical integration needs considerable time. Thus it is worth to notice that not all possible designs are different in the sense that they have different criterion values. Some of the r-point designs are only rotations, reflections or translations of other r-point designs, and therefore yield the same value of the criterion $J_{\mathcal{I}}$; let us call the respective designs 'symmetric'. To avoid calculating $J_{\mathcal{I}}$ for those designs which are known to be symmetric to others, an appropriate symmetry check can be performed before the computation of $J_{\mathcal{I}}$; for details see Gumprecht et al (2009).

3.2.2 Simple Search Algorithm

A possibility for finding a 'nearly' optimal design is the use of a simple search algorithm. This algorithm is much faster than the full enumeration algorithm as for the r-point design the number of evaluated $(r-1)$-point designs is r. This algorithm can also be performed in an acceptable time for quite large grids. The procedure is quite simple:

1. Start with a initial design ξ_0 with $S_{\xi_0} = \mathcal{X}$, called 'base' design. Thus in the first iteration the number of points r in ξ_0 is m.

2. Delete each point, one at a time, to get $(r-1)$ designs ξ_e, and compute $J_{\mathcal{I}e}$. The symmetries can be checked before the criterion is calculated.

3. Take the best $(r-1)$ design ξ_e, i.e. the design with the largest $J_{\mathcal{I}e}$, and put it as new base design.

   Go to step 2.

The algorithm stops if $r = (4+k+1)$. The r-point design that gives the largest $J_{\mathcal{I}}$ is the 'nearly' optimal one. Note the similarities to the 'coffee-house' procedure given in Müller (2007): the disadvantage of these algorithms is, that once a r-point design is chosen, all smaller $r - i$ point designs are restricted to this set of points, it can happen quite easily that one is trapped in a local maximum. To avoid this one could alternatively employ methods of stochastic optimization.

3.2.3 Fedorov Exchange Algorithm

Although nonconvexity of the criterion is an issue here, results in Gumprecht et al (2009) suggest the use of an exchange type algorithm. The 'nearly' optimal r-point design, when equal points in the design are not allowed, is found via exchanging points from it, one at a time.

1. Start with an initial r-point design, $\xi_0 = \{x_1, \ldots x_r\}$, the points are chosen at random and should be different. Compute the design criterion $J_{\mathcal{I}0}$ for the initial design.

2. Take one point x_i from ξ_0 (we call it 'base' design) and exchange it with a point not in ξ_0 - these points are called candidate points, the set of all candidate points is $\xi_c = \{\mathcal{X}|S_{\xi_0}\} = \{s_{r+1}, \ldots s_m\}$. Do this for all candidate points in ξ_c and all points in the base design ξ_0 and compute $J_{\mathcal{I}e}$ for each different combination (design). Before the criterion is computed, a symmetry check based on $diag(H)$ and λ can be performed.

3. Get the best r-point design (ξ_e), i.e. the design with the largest $J_{\mathcal{I}e}$, from the previous exchange step and put it as new base design ξ_0.

   Go to step 2.

The algorithm stops if there is no further improvement in the criterion, i.e. if $J_{\mathcal{I}e}$ is worse than $J_{\mathcal{I}}$ of the base design. In this way 'nearly' optimal r-point designs are computed for $r = 4 + k + 1, ..., m$, the overall best design is the best one of all r-point designs found by the algorithm.

4 Examples

We present two artificial and one real examples. In every case optimal designs obtained by making use of the exact distribution of Moran's $\mathcal{I}$ (i.e. $J_{\mathcal{I}}$ as a criterion) or of its normal approximation (i.e. $\tilde{J}_{\mathcal{I}}$) will be compared. For the sake of brevity we will here report only some specific instances, the full set of results can be found in the Appendix.

4.1 Artificial Examples

We will use as artificial examples two different types of structures: some regular grids and the so-called B-series from Boots and Royle (1991), a set of fourteen maximally connected planar spatial structures with a fixed number of nodes $n = 8$ and a common overall connectivity.

4.1.1 3×3, 4×4 and 5×5 grids

Since we wanted to use full enumeration we restricted attention to grids of the dimensions 3×3, 4×4 and 5×5 that consist of 9, 16 and 25 nodes respectively, taking into account rook and queen connections (see e.g. Tiefelsdorf (2000)). Considering three different values of the correlation parameter $\rho = 0.1, 0.5, 0.9$, optimal designs for these regular grids using $J_{\mathcal{I}}$ and $\tilde{J}_{\mathcal{I}}$ were obtained. Mostly the designs obtained by each method did not differ, at least with respect to the criterion value (i.e. they were symmetric by our definition). However in some cases, eg. the one depicted in Figure 1, which shows optimal designs for the 5×5-grid and an intermediate level of correlation $\rho = 0.5$, slight differences could be observed.

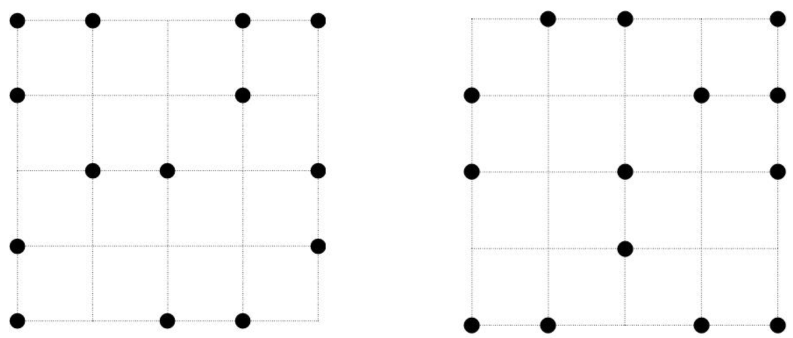

Figure 1: Optimal designs for the 5×5-grid making use of the normal approximation of the distribution of Moran's $\mathcal{I}$ (left) and of the exact distribution (right)

However, it turned out that relative efficiencies of the designs obtained using normal approximation, which can be characterized by the quotient $\max \tilde{J}_{\mathcal{I}} / \max J_{\mathcal{I}}$, are usually very high. In fact they are always much higher than 90% except for the case of a 4×4-grid with queen connections and $\rho = 0.5$ that presents an efficiency of only 88%.

It is also remarkable that, as was also already seen in Gumprecht et al (2009), the power of the test typically increases with the number of optimal observations employed up to an optimum to slightly decrease thereafter. In Figure 2, we can see the evolution of the power for the case of a 5×5-grid with rook connections, a correlation parameter $\rho = 0.5$ and designs obtained using the exact distribution. The optimum design here contains 14 nodes. Not unexpectedly the test power given by the optimal designs also increases with increasing correlation parameter ρ.

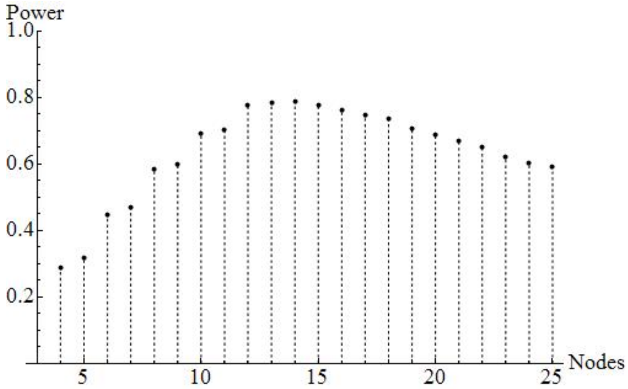

Figure 2: Power for the best designs, using the exact distribution of Moran's $\mathcal{I}$, for 5×5-grid, rook connections, $\rho = 0.5$, when the number of support points increases

4.1.2 B07 and B14 structures

The B07 and B14 structures were chosen in the attempt to highlight the differences between the use of the exact and the approximative distributions, since they were specifically designed by Boots and Royle (1991) to make the approximation fail. Considering again the three different values of the correlation parameter $\rho = 0.1, 0.5, 0.9$, we obtained optimal designs for these B-structures.

Surprisingly, the optimal designs found for the B07-structure are very similar for each of these ρ's, the test power increases with ρ and the relative efficiencies of the designs obtained using normal approximation are always equal to 1.

On the other hand, for the B14-structure, as shown in Figure 3, things are very different. For example, for $\rho = 0.5$, the optimal design obtained using $J_{\mathcal{I}}$

has 6 points in its support (crosses in Figure 3), but the design using $\tilde{J}_{\mathcal{I}}$ has only 4 points (circles in Figure 3). Furthermore we found that the power of the test decreases for high values of ρ, eventually dropping to 0, which we take as an instance of the power trap described in Krämer (2005) or Martellosio (2010) for more details. Still however, for each ρ relative efficiencies of the designs obtained using the normal approximation are greater than 97%. This indicates that the optimal design not only maximizes the power, but also leads to improving the normal approximation even in adverse settings.

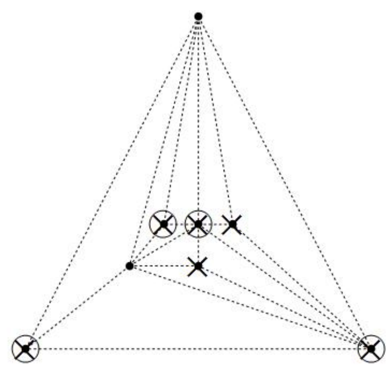

Figure 3: B14-structure. Crosses correspond to 6-points optimal design obtained using exact distribution of Moran's $\mathcal{I}$ and circles correspond to 4-points optimal design obtained using normal approximation

4.2 A real application

This section is dedicated to the application of the above described procedure to a real case of the estimation of spatial structure of species richness data. We would like to exemplify our approach at hand of a study recently undertaken by Vieira et al (2008), where the biodiversity of the Brazilian Cerrado is investigated.

Their study is very suitable for our purposes for various reasons. First they have a well defined grid of observations (see Figure 4), which can be naturally taken as our candidate set $\mathcal{X}$. Secondly, they employ a model, which is the simplest special case of (1) with $\beta = 0$. This enables us to directly use their results for defining initial values or framework conditions, required in our methods. Also the dataset is compact and handy involving only 181 observations (counts) on each birds and mammals.

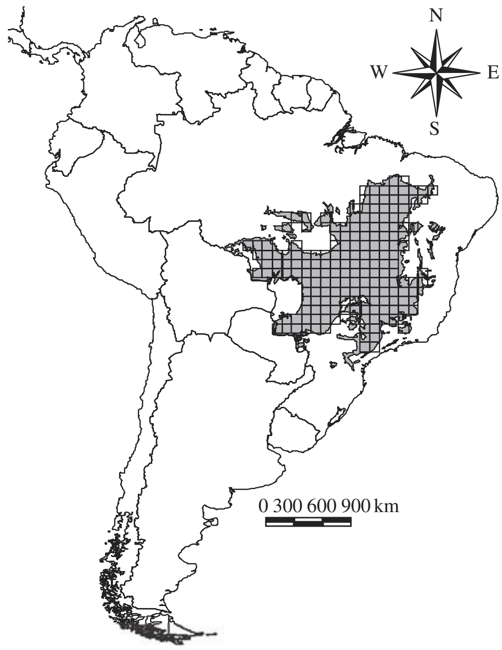

Figure 4: Sampling region in the Brazilian cerrado - this is Figure 1 of Vieira et al (2008).

In their paper Vieira et al (2008) employed different types of connections and distances between the cells (Delaunay, Gabriel, minimum spanning tree, rook, queen, etc.) and thus wanted to investigate the impact of the form of the matrix V on their spatial analysis, measured by the Akaike Information Criteria (AIC) values of their respective models. They concluded that for both birds and mammals the choice of the rook connection criterion (i.e. only to connect direct vertical and horizontal neighbours) yields the best AIC, which is why we will also concentrate on this case in what follows. They also note, however, that the choice of the type of connection is only of smaller impact as long as at least some correction for autocorrelation is undertaken. In fact our investigations on other types in the Appendix more strongly support this for the optimized designs.

Our optimized design for this setup was found by first employing the quick simple search algorithm and then refining the result by running the Fedorov

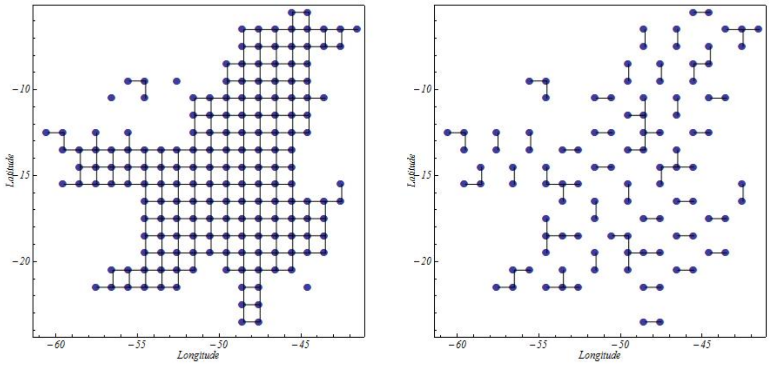

Figure 5: Optimal design for the Cerrado

exchange. Since the powers observed here were very high, particularly for high ρ's, we decided to report the case $\rho = 0.5$ only. In a practical situation, where we do not have an idea about the potential amount of correlation this might be a safe choice. Using criterion $J_{\mathcal{I}}$, this resulted in a 107 point design out of 178 candidate locations (after having deleted 3 unconnected ones), which are displayed in the right and left panel of Figure 5 respectively. The criterion $\tilde{J}_{\mathcal{I}}$ yielded a slightly different design with efficiency close to 1.

5 Conclusions

In a recent pamphlet about myths in connection with spatial autocorrelation Fortin and Dale (2009) at one place state >>[...] The context of this myth provides an excellent example of the classic advice stating that a data analytic design should be decided before sampling commences: "Think before you act!" [...]<< It was exactly this sentiment that motivated our study and eventually proved to be worthwhile. The improvement of design efficiencies we observed ranged from +17% to +165% (depending upon various settings, particularly the assumed value of ρ), thus allowing for considerable power gains for Moran's $\mathcal{I}$ test.

Furthermore it showed, somewhat surprisingly, that the optimum design acts as a kind of regulatory device making up for potential deficiencies of the normal approximation. The respective powers were always in the same range and efficiencies usually close to 100%.

Moreover we observe that the optimal designs can be rather sparse, thereby leading to vast economic improvements, especially when the sampling efforts are costly. As a side issue by our procedure we evaluate an optimal number of observations, a question that was taken up from a different angle in Griffith (2005). Finally let us note that our procedures are not restricted to the choice of Moran's $\mathcal{I}$ as the test statistic, nor to the particular choice of model (1), but is rather universally applicable for any other comparable set of circumstances.

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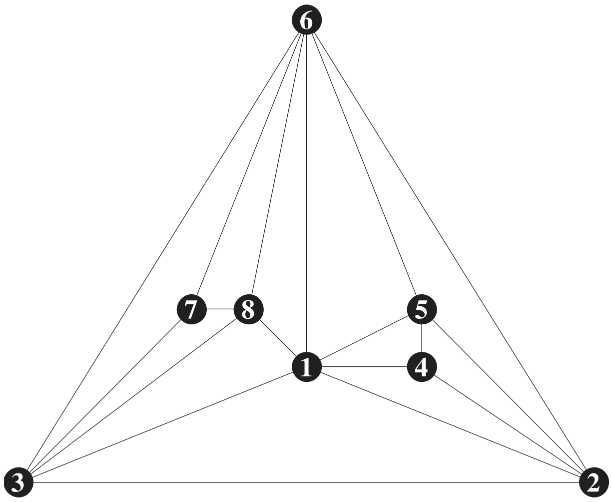
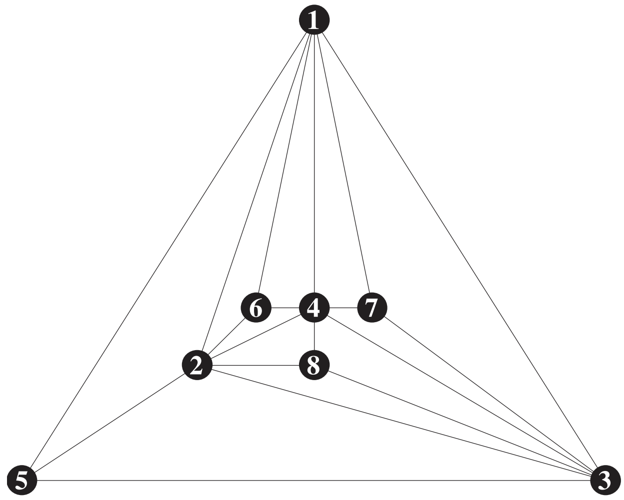
APPENDIX

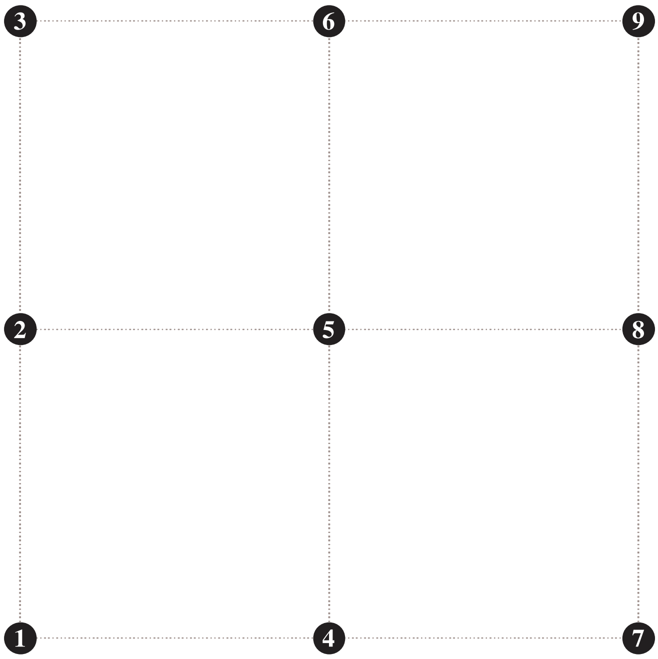
B07 – structure



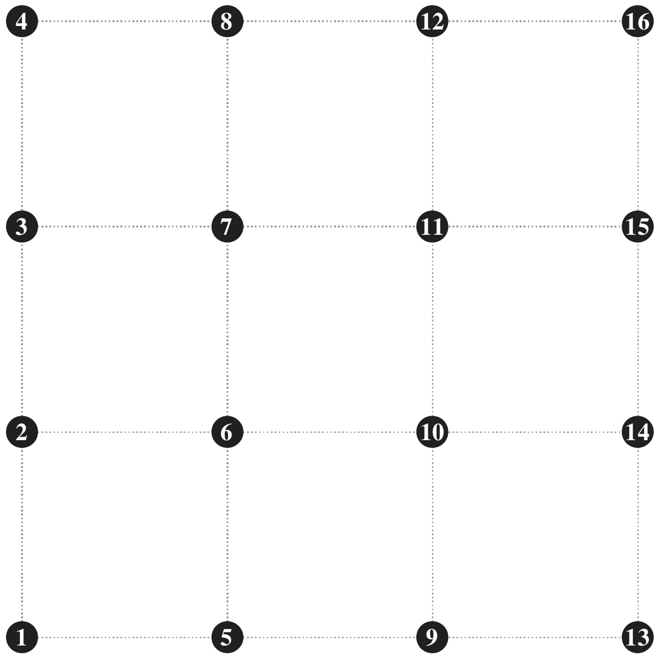
B14 – structure



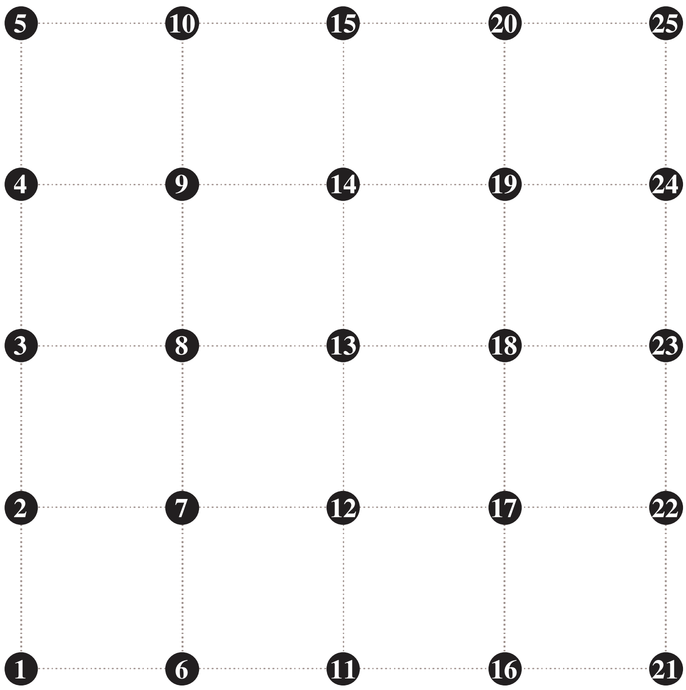
3x3 – Grid



4x4 – Grid

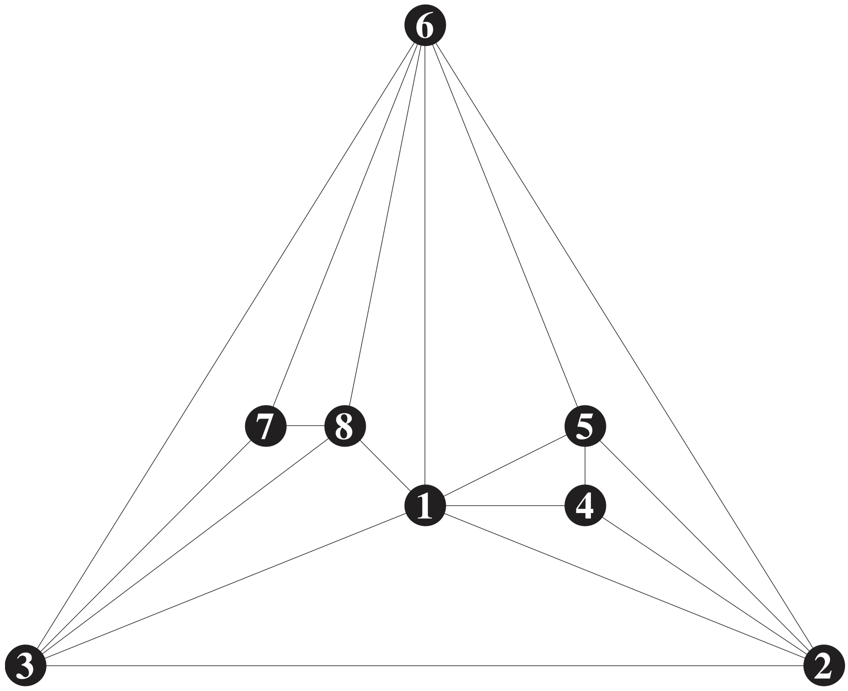


5x5 – Grid



B07
STRUCTURE

B07 – structure



B07 STRUCTURE, ρ= 0.1 , RESULTS

Normal Approximation. Optimal designs when reducing the number of points:

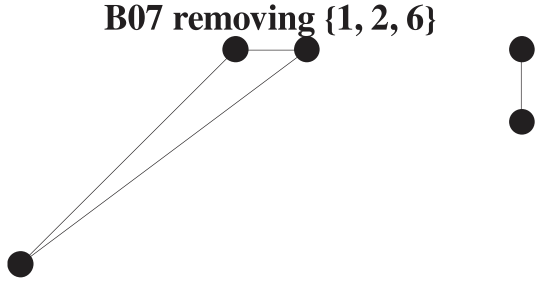
Complete Design power --> 0.07 (0.064)
Removing node/s 6 --> power 0.074 (0.067)
Removing node/s 1, 6 --> power 0.081 (0.071)
Removing node/s 1, 3, 6 --> power 0.085 (0.075)
Removing node/s 1, 2, 6, 7 --> power 0.082 (0.072)
Optimal Design (normal approx.) when removing node/s 1, 3, 6 with (normal) power 0.085 (0.075)

Exact Distribution. Optimal designs when reducing the number of points:

Complete Design power --> 0.064
Removing node/s 1 --> power 0.067
Removing node/s 1, 6 --> power 0.071
Removing node/s 1, 2, 6 --> power 0.075
Removing node/s 1, 2, 3, 6 --> power 0.072
Optimal Design (exact) when removing node/s 1, 2, 6 with power 0.075

Efficiency of the normal-optimal respect to the exact-optimal: 0.075 / 0.075 = 1

B07 removing {1, 2, 6}



B07 STRUCTURE, ρ= 0.5 , RESULTS

Normal Approximation. Optimal designs when reducing the number of points:

Complete Design power --> 0.194 (0.158)
Removing node/s 6 --> power 0.228 (0.189)
Removing node/s 1, 6 --> power 0.286 (0.246)
Removing node/s 1, 2, 6 --> power 0.328 (0.318)
Removing node/s 1, 2, 6, 7 --> power 0.281 (0.288)
Optimal Design (normal approx.) when removing node/s 1, 2, 6 with (normal) power 0.328 (0.318)

Exact Distribution. Optimal designs when reducing the number of points:

Complete Design power --> 0.158
Removing node/s 1 --> power 0.189
Removing node/s 1, 6 --> power 0.246
Removing node/s 1, 2, 6 --> power 0.318
Removing node/s 1, 2, 3, 6 --> power 0.288
Optimal Design (exact) when removing node/s 1, 2, 6 with power 0.318

Efficiency of the normal-optimal respect to the exact-optimal: 0.318 / 0.318 = 1

B07 STRUCTURE, ρ= 0.9 , RESULTS

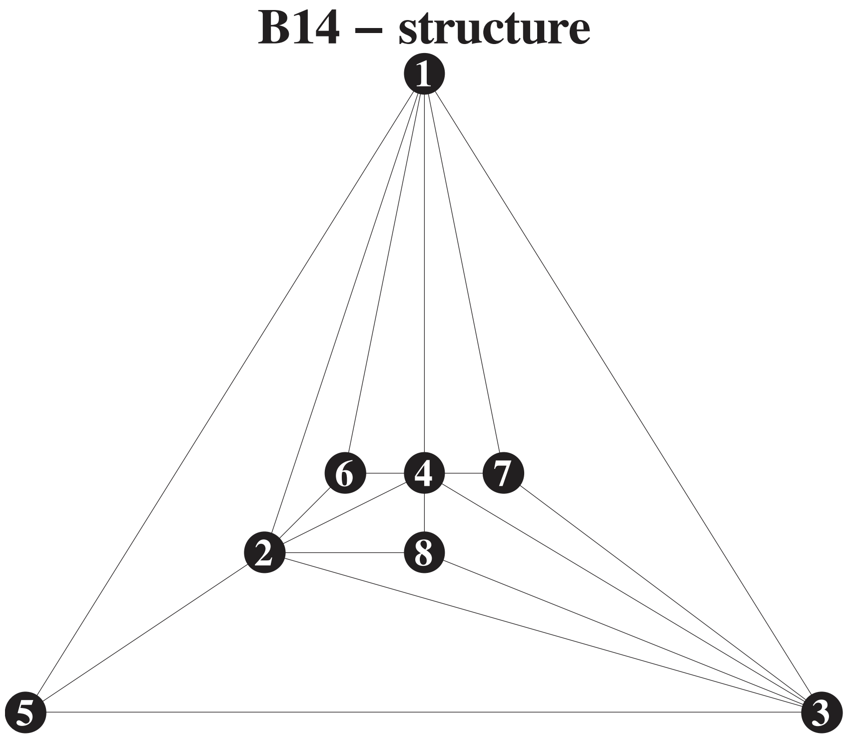
Normal Approximation. Optimal designs when reducing the number of points:

Complete Design power --> 0.265 (0.322)
Removing node/s 6 --> power 0.36 (0.417)
Removing node/s 1, 6 --> power 0.534 (0.605)
Removing node/s 1, 3, 6 --> power 0.831 (0.854)
Removing node/s 1, 2, 6, 7 --> power 0.701 (0.842)
Optimal Design (normal approx.) when removing node/s 1, 3, 6 with (normal) power 0.831 (0.854)

Exact Distribution. Optimal designs when reducing the number of points:

Complete Design power --> 0.322
Removing node/s 1 --> power 0.417
Removing node/s 1, 6 --> power 0.605
Removing node/s 1, 2, 6 --> power 0.854
Removing node/s 1, 2, 6, 7 --> power 0.842
Optimal Design (exact) when removing node/s 1, 2, 6 with power 0.854

Efficiency of the normal-optimal respect to the exact-optimal: 0.854 / 0.854 = 1



B14
STRUCTURE

B14 STRUCTURE, ρ= 0.1 , RESULTS

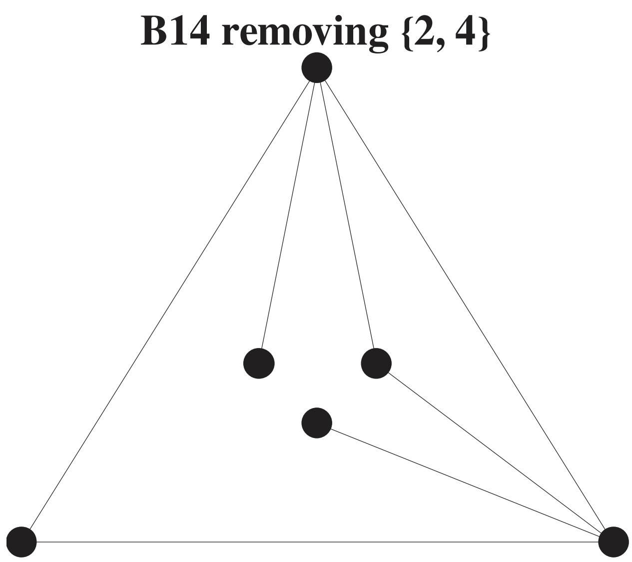
Normal Approximation. Optimal designs when reducing the number of points:

Complete Design power --> 0.061 (0.06)
Removing node/s 4 --> power 0.062 (0.061)
Removing node/s 2, 4 --> power 0.065 (0.065)
Removing node/s 1, 3, 8 --> power 0.067 (0.064)
Removing node/s 1, 2, 7, 8 --> power 0.067 (0.063)
Optimal Design (normal approx.) when removing node/s 1, 2, 7, 8 with (normal) power 0.067 (0.063)

Exact Distribution. Optimal designs when reducing the number of points:

Complete Design power --> 0.06
Removing node/s 1 --> power 0.061
Removing node/s 2, 4 --> power 0.065
Removing node/s 1, 2, 7 --> power 0.064
Removing node/s 1, 2, 7, 8 --> power 0.063
Optimal Design (exact) when removing node/s 2, 4 with power 0.065

Efficiency of the normal-optimal respect to the exact-optimal: 0.063 / 0.065 = 0.9824



B14 STRUCTURE, ρ= 0.5 , RESULTS

Normal Approximation. Optimal designs when reducing the number of points:

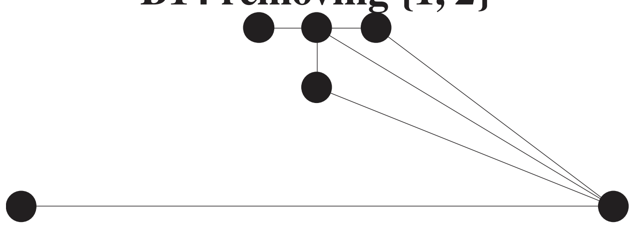
Complete Design power --> 0.109 (0.113)
Removing node/s 1 --> power 0.111 (0.124)
Removing node/s 1, 8 --> power 0.124 (0.124)
Removing node/s 1, 2, 7 --> power 0.134 (0.151)
Removing node/s 1, 2, 7, 8 --> power 0.144 (0.148)
Optimal Design (normal approx.) when removing node/s 1, 2, 7, 8 with (normal) power 0.144 (0.148)

Exact Distribution. Optimal designs when reducing the number of points:

Complete Design power --> 0.113
Removing node/s 2 --> power 0.124
Removing node/s 1, 2 --> power 0.151
Removing node/s 1, 2, 7 --> power 0.151
Removing node/s 1, 2, 7, 8 --> power 0.148
Optimal Design (exact) when removing node/s 1, 2 with power 0.151

Efficiency of the normal-optimal respect to the exact-optimal: 0.148 / 0.151 = 0.97565

B14 removing {1, 2}



B14 STRUCTURE, ρ= 0.9 , RESULTS

Normal Approximation. Optimal designs when reducing the number of points:

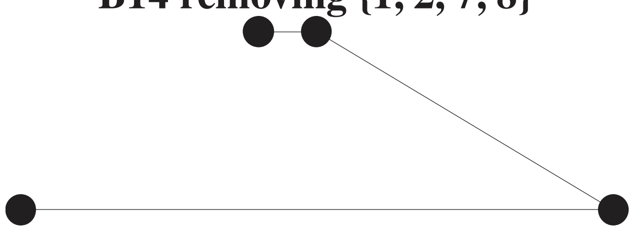
Complete Design power --> 0.044 (0.192)
Removing node/s 7 --> power 0.049 (0.172)
Removing node/s 5, 7 --> power 0.056 (0.145)
Removing node/s 1, 6, 8 --> power 0.045 (0.183)
Removing node/s 1, 2, 7, 8 --> power 0.062 (0.292)
Optimal Design (normal approx.) when removing node/s 1, 2, 7, 8 with (normal) power 0.062 (0.292)

Exact Distribution. Optimal designs when reducing the number of points:

Complete Design power --> 0.192
Removing node/s 3 --> power 0.219
Removing node/s 1, 4 --> power 0.276
Removing node/s 1, 4, 8 --> power 0.283
Removing node/s 1, 2, 7, 8 --> power 0.292
Optimal Design (exact) when removing node/s 1, 2, 7, 8 with power 0.292

Efficiency of the normal-optimal respect to the exact-optimal: 0.292 / 0.292 = 1

B14 removing {1, 2, 7, 8}



SIMPLE SEARCH ALGORITHM FOR B14 STRUCTURE AND NORMAL APPROX, , ρ= 0.5

```
[1] Complete design power: 0.108827434294412 ( 0.113194866001931 )
 [1] * Best 7 -point design when removing node 1 with power 0.110983931952286 ( 0.124013031860449 )
[1] * Best 6 -point design when removing node 7 with power 0.124261987513663 ( 0.123726605878900 )
[1] * Best 5 -point design when removing node 1 with power 0.133867708650032 ( 0.150585320604468 )
[1] * Best 4 -point design when removing node 5 with power 0.144496530519374 ( 0.147593110580022 )
[1] * Best 3 -point design when removing node 1 with power 0 ( 0 )
> powers
[1] 0.1109839 0.1242620 0.1338677 0.1444965 0.0000000 0.0000000 0.1088274
> max(powers)
[1] 0.1444965
> which.max(powers)
[1] 4
> removed
[1] 1 8 2 7 3 4
```

SIMPLE SEARCH ALGORITHM FOR B14 STRUCTURE AND NORMAL APPROX, , ρ= 0.9

```
[1] Complete design power: 0.0441798469324515 ( 0.192197884516487 )
 [1] * Best 7 -point design when removing node 7 with power 0.0492200303834604 ( 0.171572717906891 )
[1] * Best 6 -point design when removing node 5 with power 0.0556311850168232 ( 0.145199365556077 )
[1] * Best 5 -point design when removing node 4 with power 0.044920569629419 ( 0.182837227149421 )
[1] * Best 4 -point design when removing node 2 with power 0.062242499914308 ( 0.292318988543839 )
[1] * Best 3 -point design when removing node 1 with power 0 ( 0 )
> powers
[1] 0.04922003 0.05563119 0.04492057 0.06224250 0.00000000 0.00000000 0.04417985
> max(powers) 0.0622425
> which.max(powers)  4
```

SIMPLE SEARCH ALGORITHM FOR B14 STRUCTURE AND NORMAL APPROX, , ρ= 0.921

```
 [1] Complete design power: 0.0294064963192422 ( 0.197060343508010 )
[1] * Best 7 -point design when removing node 5 with power 0.0334196336617642 ( 0.175615487080113 )
[1] * Best 6 -point design when removing node 5 with power 0.0391712943026559 ( 0.148190654485884 )
[1] * Best 5 -point design when removing node 3 with power 0.0286606179621427 ( 0.187341448411514 )
[1] * Best 4 -point design when removing node 3 with power 0.0414112330181991 ( 0.301679559998708 )
[1] * Best 3 -point design when removing node 1 with power 0 ( 0 )
> powers
[1] 0.03341963 0.03917129 0.02866062 0.04141123 0.00000000 0.00000000 0.02940650
> max(powers)  0.04141123
> which.max(powers)  4
```

SIMPLE SEARCH ALGORITHM FOR B14 STRUCTURE AND NORMAL APPROX, , ρ= 1-0.921

```
[1] Complete design power: 0.0588620734808486 ( 0.0575999197891122 )
[1] * Best 7 -point design when removing node 1 with power 0.059686929463195 ( 0.0586553753454684 )
[1] * Best 6 -point design when removing node 3 with power 0.0615730566011281 ( 0.061257424865933 )
[1] * Best 5 -point design when removing node 6 with power 0.0629219406114719 ( 0.0609311570370701 )
[1] * Best 4 -point design when removing node 3 with power 0.0635848777588228 ( 0.0603787186059464 )
[1] * Best 3 -point design when removing node 1 with power 0 ( 0 )
> powers
[1] 0.05968693 0.06157306 0.06292194 0.06358488 0.00000000 0.00000000 0.05886207
> max(powers)  0.06358488
> which.max(powers) 4
```

```
[1] B14 STRUCTURE, rho= 0.8 , RESULTS
[1]
[1] Normal Approximation. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power     -->  0.094
[1] Removing node/s 5 --> power 0.099
[1] Removing node/s 1, 8 --> power 0.104
[1] Removing node/s 1, 7, 8 --> power 0.101
[1] Removing node/s 1, 2, 7, 8 --> power 0.132
[1] Removing node/s 1, 2, 3, 4, 5 --> power 0
[1] Removing node/s 1, 2, 3, 4, 5, 6 --> power 0
[1] Optimal Design (normal approx.) when removing node/s 1, 2, 7, 8  with power 0.132
[1]
[1]
[1] Exact Distribution. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power     -->  0.17
[1] Removing node/s 1 --> power 0.192
[1] Removing node/s 2, 3 --> power 0.242
[1] Removing node/s 1, 2, 7 --> power 0.246
[1] Removing node/s 1, 2, 7, 8 --> power 0.25
[1] Removing node/s 1, 2, 3, 4, 5 --> power 0
[1] Removing node/s 1, 2, 3, 4, 5, 6 --> power 0
[1] Optimal Design (exact) when removing node/s 1, 2, 7, 8  with power 0.25


[1] B14 STRUCTURE, rho= 0.85 , RESULTS
[1]
[1] Normal Approximation. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power     -->  0.074
[1] Removing node/s 5 --> power 0.08
[1] Removing node/s 5, 7 --> power 0.085
[1] Removing node/s 1, 5, 8 --> power 0.078
[1] Removing node/s 1, 2, 7, 8 --> power 0.104
[1] Removing node/s 1, 2, 3, 4, 5 --> power 0
[1] Removing node/s 1, 2, 3, 4, 5, 6 --> power 0
[1] Optimal Design (normal approx.) when removing node/s 1, 2, 7, 8  with power 0.104
[1]
[1]
[1] Exact Distribution. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power     -->  0.181
[1] Removing node/s 1 --> power 0.205
[1] Removing node/s 2, 3 --> power 0.259
[1] Removing node/s 1, 4, 8 --> power 0.265
[1] Removing node/s 1, 2, 7, 8 --> power 0.271
[1] Removing node/s 1, 2, 3, 4, 5 --> power 0
[1] Removing node/s 1, 2, 3, 4, 5, 6 --> power 0
[
```

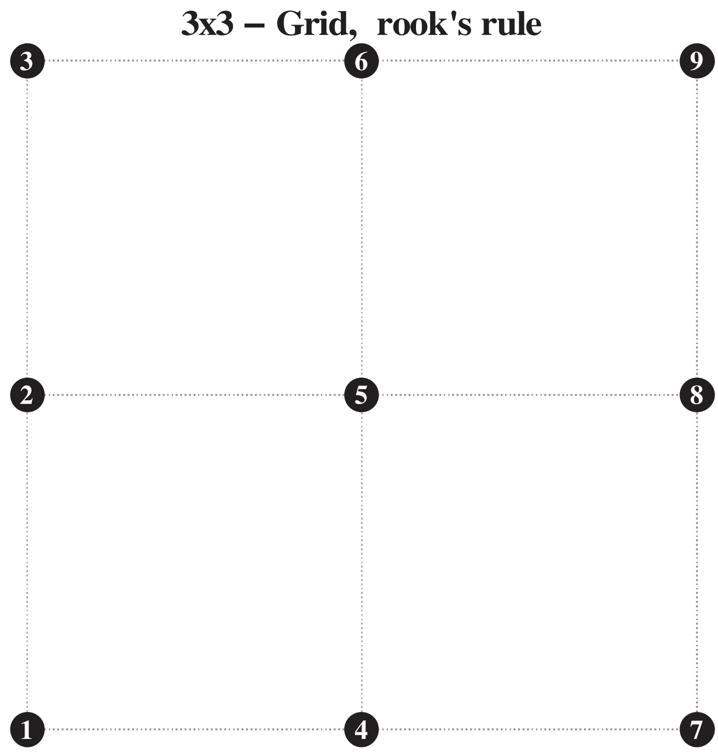
```
[1] B14 STRUCTURE, rho= 0.89 , RESULTS
[1]
[1] Normal Approximation. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power     -->  0.051
[1] Removing node/s 5 --> power 0.056
[1] Removing node/s 5, 7 --> power 0.063
[1] Removing node/s 1, 6, 8 --> power 0.052
[1] Removing node/s 1, 2, 7, 8 --> power 0.072
[1] Removing node/s 1, 2, 3, 4, 5 --> power 0
[1] Removing node/s 1, 2, 3, 4, 5, 6 --> power 0
[1] Optimal Design (normal approx.) when removing node/s 1, 2, 7, 8  with power 0.072
[1]
[1]
[1] Exact Distribution. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power     -->  0.19
[1] Removing node/s 2 --> power 0.216
[1] Removing node/s 3, 4 --> power 0.272
[1] Removing node/s 1, 2, 7 --> power 0.279
[1] Removing node/s 1, 2, 7, 8 --> power 0.288
[1] Removing node/s 1, 2, 3, 4, 5 --> power 0
[1] Removing node/s 1, 2, 3, 4, 5, 6 --> power 0
[1] Optimal Design (exact) when removing node/s 1, 2, 7, 8  with power 0.288


[1] B14 STRUCTURE, rho= 0.9 , RESULTS
[1]
[1] Normal Approximation. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power     -->  0.044
[1] Removing node/s 7 --> power 0.049
[1] Removing node/s 5, 7 --> power 0.056
[1] Removing node/s 1, 6, 8 --> power 0.045
[1] Removing node/s 1, 2, 7, 8 --> power 0.062
[1] Removing node/s 1, 2, 3, 4, 5 --> power 0
[1] Removing node/s 1, 2, 3, 4, 5, 6 --> power 0
[1] Optimal Design (normal approx.) when removing node/s 1, 2, 7, 8  with power 0.062
[1]
[1]
[1] Exact Distribution. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power     -->  0.192
[1] Removing node/s 3 --> power 0.219
[1] Removing node/s 1, 4 --> power 0.276
[1] Removing node/s 1, 4, 8 --> power 0.283
[1] Removing node/s 1, 2, 7, 8 --> power 0.292
[1] Removing node/s 1, 2, 3, 4, 5 --> power 0
[1] Removing node/s 1, 2, 3, 4, 5, 6 --> power 0
[1] Optimal Design (exact) when removing node/s 1, 2, 7, 8  with power 0.292

>
```

[1] B14 STRUCTURE, rho= 0.91 , RESULTS
[1]
[1] Normal Approximation. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power --> 0.037
[1] Removing node/s 5 --> power 0.042
[1] Removing node/s 5, 6 --> power 0.048
[1] Removing node/s 1, 6, 8 --> power 0.037
[1] Removing node/s 1, 2, 7, 8 --> power 0.052
[1] Removing node/s 1, 2, 3, 4, 5 --> power 0
[1] Removing node/s 1, 2, 3, 4, 5, 6 --> power 0
[1] Optimal Design (normal approx.) when removing node/s 1, 2, 7, 8 with power 0.052
[1]
[1]
[1] Exact Distribution. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power --> 0.195
[1] Removing node/s 2 --> power 0.221
[1] Removing node/s 1, 4 --> power 0.279
[1] Removing node/s 1, 4, 8 --> power 0.287
[1] Removing node/s 1, 2, 7, 8 --> power 0.297
[1] Removing node/s 1, 2, 3, 4, 5 --> power 0
[1] Removing node/s 1, 2, 3, 4, 5, 6 --> power 0
[1] Optimal Design (exact) when removing node/s 1, 2, 7, 8 with power 0.297

[1] B14 STRUCTURE, rho= 0.95 , RESULTS
[1]
[1] Normal Approximation. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power --> 0.01
[1] Removing node/s 6 --> power 0.012
[1] Removing node/s 6, 7 --> power 0.015
[1] Removing node/s 1, 6, 7 --> power 0.009
[1] Removing node/s 1, 2, 7, 8 --> power 0.014
[1] Removing node/s 1, 2, 3, 4, 5 --> power 0
[1] Removing node/s 1, 2, 3, 4, 5, 6 --> power 0
[1] Optimal Design (normal approx.) when removing node/s 6, 7 with power 0.015
[1]
[1]
[1] Exact Distribution. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power --> 0.204
[1] Removing node/s 3 --> power 0.232
[1] Removing node/s 1, 3 --> power 0.293
[1] Removing node/s 1, 2, 7 --> power 0.302
[1] Removing node/s 1, 2, 7, 8 --> power 0.315
[1] Removing node/s 1, 2, 3, 4, 5 --> power 0
[1] Removing node/s 1, 2, 3, 4, 5, 6 --> power 0
[1] Optimal Design (exact) when removing node/s 1, 2, 7, 8 with power 0.315

[1] B14 STRUCTURE, rho= 0.99 , RESULTS
[1]
[1] Normal Approximation. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power --> 0
[1] Removing node/s 8 --> power 0
[1] Removing node/s 5, 6 --> power 0
[1] Removing node/s 1, 6, 7 --> power 0
[1] Removing node/s 1, 2, 7, 8 --> power 0
[1] Removing node/s 1, 2, 3, 4, 5 --> power 0
[1] Removing node/s 1, 2, 3, 4, 5, 6 --> power 0
[1] Optimal Design (normal approx.) when removing node/s 5, 6 with power 0
[1]
[1]
[1] Exact Distribution. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power --> 0.214
[1] Removing node/s 2 --> power 0.244
[1] Removing node/s 3, 4 --> power 0.307
[1] Removing node/s 1, 3, 8 --> power 0.318
[1] Removing node/s 1, 2, 7, 8 --> power 0.334
[1] Removing node/s 1, 2, 3, 4, 5 --> power 0
[1] Removing node/s 1, 2, 3, 4, 5, 6 --> power 0
[1] Optimal Design (exact) when removing node/s 1, 2, 7, 8 with power 0.334

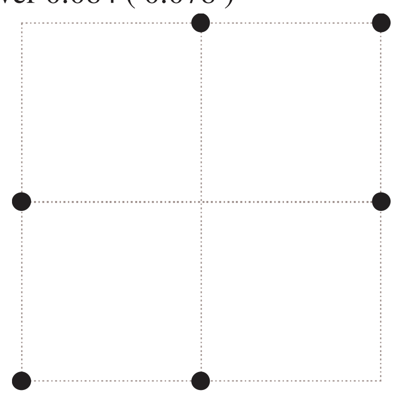
GRID 3x3
ROOK'S RULE



GRID 3 x 3 ROOK'S RULE, ρ= 0.1 , RESULTS

Normal Approximation. Optimal designs when reducing the number of points:

Complete Design power --> 0.071 (0.072)
Removing node/s 5 --> power 0.077 (0.076)
Removing node/s 1, 5 --> power 0.08 (0.076)
Removing node/s 1, 5, 6 --> power 0.084 (0.078)
Removing node/s 1, 4, 5, 9 --> power 0.082 (0.075)
Removing node/s 1, 2, 3, 5, 8 --> power 0.082 (0.072)
Optimal Design (normal approx.) when removing node/s 1, 5, 6 with (normal) power 0.084 (0.078)



Exact Distribution. Optimal designs when reducing the number of points:

Complete Design power --> 0.072
Removing node/s 5 --> power 0.076
Removing node/s 5, 6 --> power 0.076
Removing node/s 3, 5, 7 --> power 0.078
Removing node/s 1, 2, 5, 9 --> power 0.075
Removing node/s 1, 2, 3, 5, 8 --> power 0.072
Optimal Design (exact) when removing node/s 3, 5, 7 with power 0.078

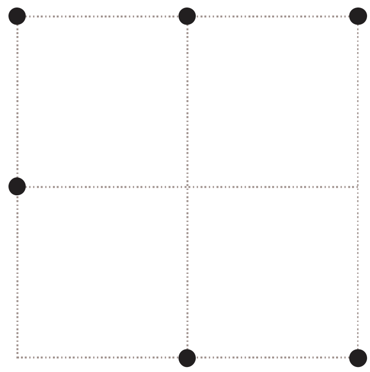
Efficiency of the normal-optimal respect to the exact-optimal: 0.078 / 0.078 = 0.99941

GRID 3 x 3 ROOK'S RULE, ρ= 0.5 , RESULTS

Normal Approximation. Optimal designs when reducing the number of points:

Complete Design power --> 0.213 (0.243)
Removing node/s 5 --> power 0.272 (0.298)
Removing node/s 3, 5 --> power 0.285 (0.307)
Removing node/s 3, 5, 8 --> power 0.314 (0.345)
Removing node/s 1, 5, 8, 9 --> power 0.287 (0.319)
Removing node/s 1, 2, 3, 5, 8 --> power 0.281 (0.288)
Optimal Design (normal approx.) when removing node/s 3, 5, 8 with (normal) power 0.314 (0.345)

Exact Distribution. Optimal designs when reducing the number of points:



Complete Design power --> 0.243
Removing node/s 5 --> power 0.298
Removing node/s 5, 8 --> power 0.307
Removing node/s 1, 5, 8 --> power 0.345
Removing node/s 3, 5, 6, 7 --> power 0.319
Removing node/s 1, 2, 3, 5, 8 --> power 0.288
Optimal Design (exact) when removing node/s 1, 5, 8 with power 0.345

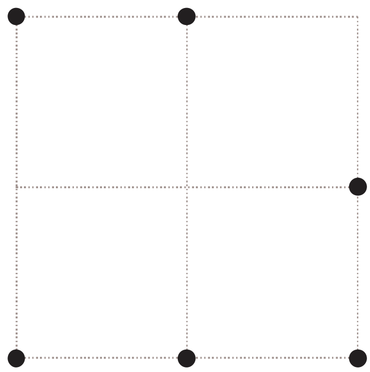
Efficiency of the normal-optimal respect to the exact-optimal: 0.345 / 0.345 = 1

GRID 3 x 3 ROOK'S RULE, ρ= 0.9 , RESULTS

Normal Approximation. Optimal designs when reducing the number of points:

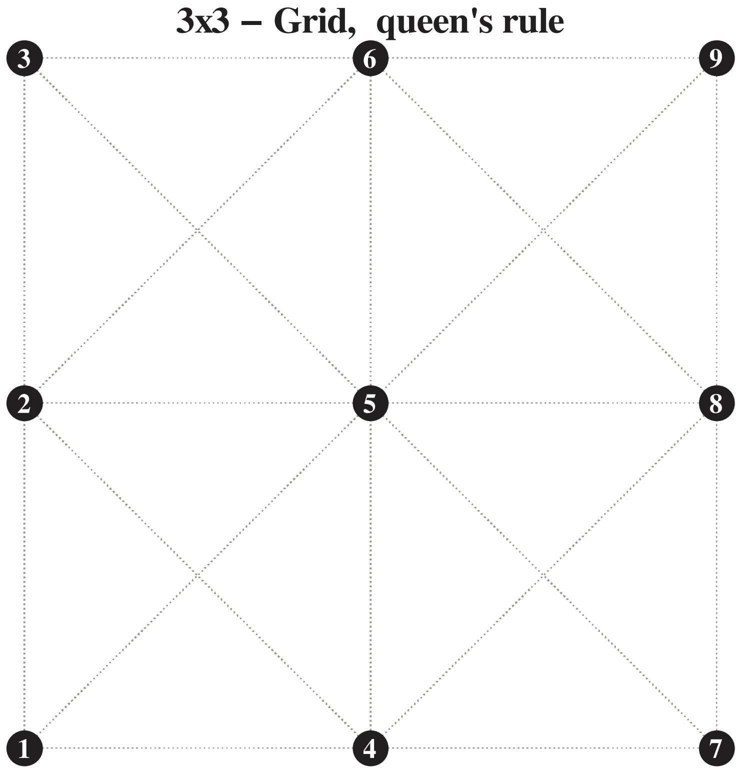
Complete Design power --> 0.331 (0.55)
Removing node/s 5 --> power 0.631 (0.702)
Removing node/s 3, 5 --> power 0.485 (0.732)
Removing node/s 3, 5, 8 --> power 0.638 (0.862)
Removing node/s 2, 3, 5, 7 --> power 0.512 (0.848)
Removing node/s 1, 2, 4, 5, 9 --> power 0.701 (0.842)
Optimal Design (normal approx.) when removing node/s 1, 2, 4, 5, 9 with (normal) power 0.701 (0.842)

Exact Distribution. Optimal designs when reducing the number of points:



Complete Design power --> 0.55
Removing node/s 5 --> power 0.702
Removing node/s 4, 5 --> power 0.732
Removing node/s 2, 5, 9 --> power 0.862
Removing node/s 2, 3, 5, 7 --> power 0.848
Removing node/s 1, 2, 3, 5, 8 --> power 0.842
Optimal Design (exact) when removing node/s 2, 5, 9 with power 0.862

Efficiency of the normal-optimal respect to the exact-optimal: 0.842 / 0.862 = 0.97648

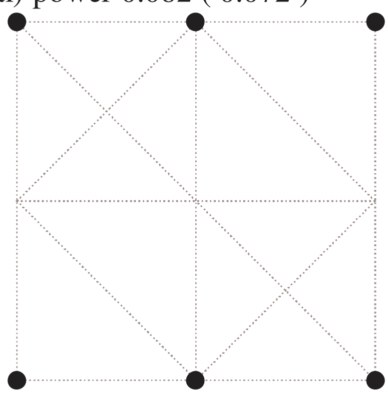


GRID 3x3
QUEEN'S RULE

GRID 3 x 3 QUEEN'S RULE, ρ= 0.1 , RESULTS

Normal Approximation. Optimal designs when reducing the number of points:

Complete Design power --> 0.068 (0.064)
Removing node/s 5 --> power 0.074 (0.069)
Removing node/s 2, 5 --> power 0.076 (0.072)
Removing node/s 4, 5, 6 --> power 0.082 (0.078)
Removing node/s 4, 5, 6, 7 --> power 0.082 (0.075)
Removing node/s 1, 2, 3, 5, 8 --> power 0.082 (0.072)
Optimal Design (normal approx.) when removing node/s 1, 2, 3, 5, 8 with (normal) power 0.082 (0.072)



Exact Distribution. Optimal designs when reducing the number of points:

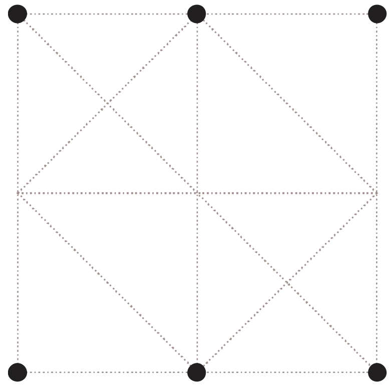
Complete Design power --> 0.064
Removing node/s 5 --> power 0.069
Removing node/s 2, 5 --> power 0.072
Removing node/s 2, 5, 8 --> power 0.078
Removing node/s 1, 4, 5, 6 --> power 0.075
Removing node/s 1, 2, 3, 5, 8 --> power 0.072
Optimal Design (exact) when removing node/s 2, 5, 8 with power 0.078

Efficiency of the normal-optimal respect to the exact-optimal: 0.072 / 0.078 = 0.92371

GRID 3 x 3 QUEEN'S RULE, ρ= 0.5 , RESULTS

Normal Approximation. Optimal designs when reducing the number of points:

Complete Design power   --> 0.164 (0.157)
Removing node/s 5 --> power 0.214 (0.211)
Removing node/s 2, 5 --> power 0.232 (0.241)
Removing node/s 2, 5, 8 --> power 0.301 (0.338)
Removing node/s 2, 3, 5, 8 --> power 0.287 (0.319)
Removing node/s 1, 2, 3, 5, 8 --> power 0.281 (0.288)
Optimal Design (normal approx.) when removing node/s 2, 5, 8 with (normal) power 0.301 (0.338)



Exact Distribution. Optimal designs when reducing the number of points:

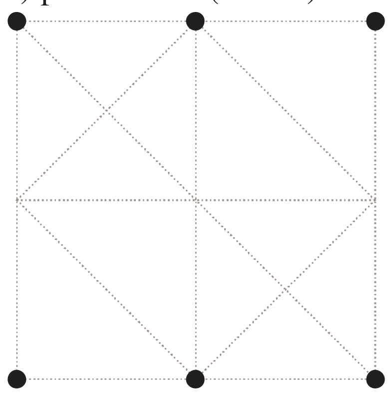
Complete Design power   --> 0.157
Removing node/s 5 --> power 0.211
Removing node/s 2, 5 --> power 0.241
Removing node/s 2, 5, 8 --> power 0.338
Removing node/s 1, 4, 5, 6 --> power 0.319
Removing node/s 1, 2, 3, 5, 8 --> power 0.288
Optimal Design (exact) when removing node/s 2, 5, 8 with power 0.338

Efficiency of the normal-optimal respect to the exact-optimal: 0.338 / 0.338 = 1

GRID 3 x 3 QUEEN'S RULE, ρ= 0.9 , RESULTS

Normal Approximation. Optimal designs when reducing the number of points:

Complete Design power   --> 0.098 (0.315)
Removing node/s 5 --> power 0.171 (0.466)
Removing node/s 2, 5 --> power 0.228 (0.537)
Removing node/s 4, 5, 6 --> power 0.507 (0.853)
Removing node/s 2, 5, 7, 8 --> power 0.512 (0.848)
Removing node/s 1, 4, 5, 6, 7 --> power 0.701 (0.842)
Optimal Design (normal approx.) when removing node/s 1, 4, 5, 6, 7 with (normal) power 0.701 (0.842)

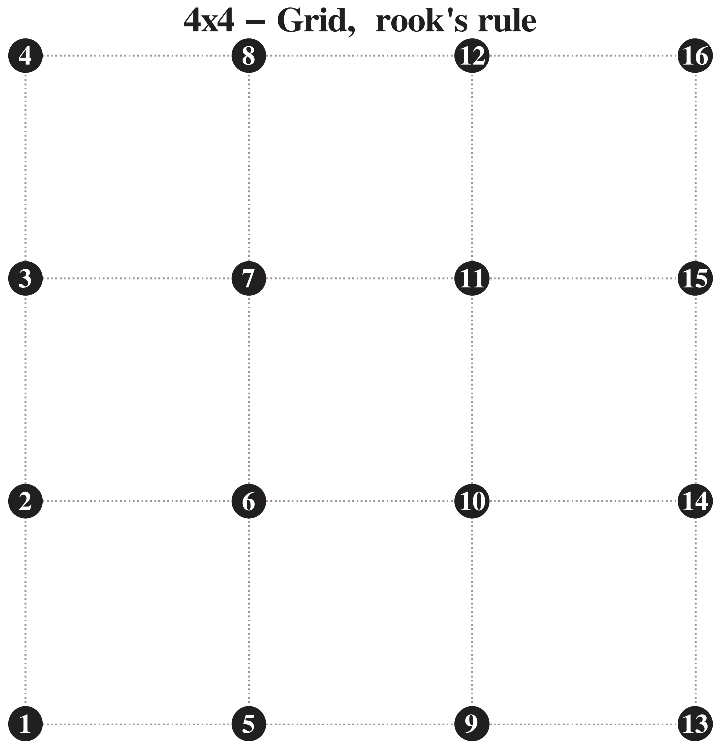


Exact Distribution. Optimal designs when reducing the number of points:

Complete Design power   --> 0.315
Removing node/s 5 --> power 0.466
Removing node/s 2, 5 --> power 0.537
Removing node/s 2, 5, 8 --> power 0.853
Removing node/s 1, 2, 5, 8 --> power 0.848
Removing node/s 1, 2, 3, 5, 8 --> power 0.842
Optimal Design (exact) when removing node/s 2, 5, 8 with power 0.853

Efficiency of the normal-optimal respect to the exact-optimal: 0.842 / 0.853 = 0.98696

GRID 4x4
ROOK'S RULE



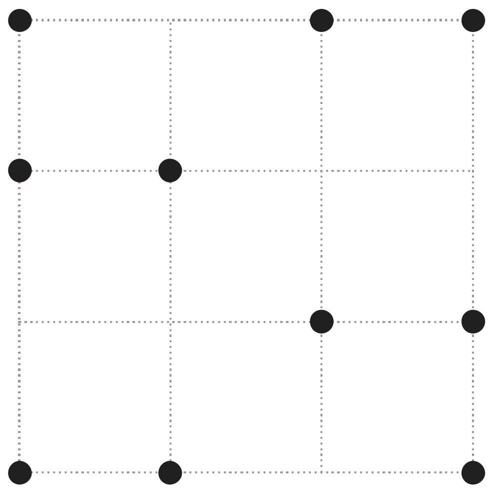
GRID 4 x 4 ROOK'S RULE, ρ= 0.1 , RESULTS

Normal Approximation. Optimal designs when reducing the number of points:

Complete Design power --> 0.085 (0.086)
Removing node/s 6 --> power 0.087 (0.087)
Removing node/s 11, 14 --> power 0.09 (0.089)
Removing node/s 2, 7, 10 --> power 0.093 (0.092)
Removing node/s 5, 7, 10, 15 --> power 0.096 (0.094)
Removing node/s 2, 6, 8, 9, 11 --> power 0.098 (0.096)
Removing node/s 2, 6, 8, 9, 11, 15 --> power 0.101 (0.097)
Removing node/s 2, 6, 8, 9, 10, 11, 15 --> power 0.1 (0.095)
Removing node/s 1, 2, 7, 8, 9, 10, 15, 16 --> power 0.099 (0.093)
Removing node/s 1, 2, 3, 4, 7, 9, 10, 15, 16 --> power 0.092 (0.086)
Removing node/s 1, 2, 3, 4, 6, 7, 10, 11, 13, 16 --> power 0.091 (0.083)
Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 10, 15, 16 --> power 0.082 (0.075)
Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15 --> power 0.082 (0.072)
Optimal Design (normal approx.) when removing node/s 2, 6, 8, 9, 11, 15 with (normal) power 0.101 (0.097)

Exact Distribution. Optimal designs when reducing the number of points:

Complete Design power --> 0.086
Removing node/s 7 --> power 0.087
Removing node/s 3, 6 --> power 0.089
Removing node/s 5, 7, 10 --> power 0.092
Removing node/s 3, 6, 9, 11 --> power 0.094
Removing node/s 5, 7, 10, 12, 14 --> power 0.096
Removing node/s 2, 6, 8, 9, 11, 15 --> power 0.097
Removing node/s 2, 6, 7, 8, 9, 11, 15 --> power 0.095
Removing node/s 1, 3, 5, 7, 10, 12, 14, 16 --> power 0.093
Removing node/s 1, 2, 4, 5, 8, 10, 11, 14, 15 --> power 0.086
Removing node/s 1, 2, 3, 4, 5, 8, 9, 10, 11, 15 --> power 0.083
Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 14 --> power 0.075
Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 15 --> power 0.072
Optimal Design (exact) when removing node/s 2, 6, 8, 9, 11, 15 with power 0.097



Efficiency of the normal-optimal respect to the exact-optimal: 0.097 / 0.097 = 1

GRID 4 x 4 ROOK'S RULE, ρ= 0.5 , RESULTS

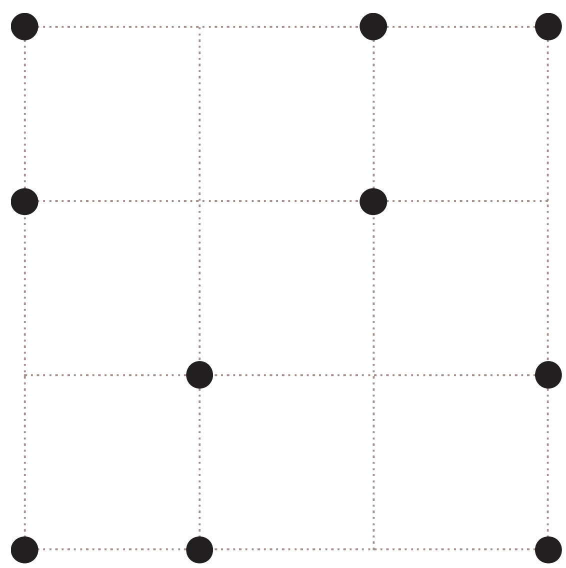
Normal Approximation. Optimal designs when reducing the number of points:

Complete Design power   --> 0.408 (0.422)
Removing node/s 6 --> power 0.426 (0.442)
Removing node/s 7, 10 --> power 0.449 (0.47)
Removing node/s 2, 6, 9 --> power 0.478 (0.499)
Removing node/s 2, 6, 9, 11 --> power 0.493 (0.532)
Removing node/s 5, 6, 11, 12, 14 --> power 0.508 (0.553)
Removing node/s 2, 6, 8, 9, 11, 15 --> power 0.534 (0.612)
Removing node/s 3, 5, 7, 10, 12, 14, 16 --> power 0.511 (0.599)
Removing node/s 1, 2, 7, 8, 9, 10, 15, 16 --> power 0.487 (0.584)
Removing node/s 1, 2, 3, 5, 6, 7, 10, 12, 14 --> power 0.398 (0.471)
Removing node/s 1, 2, 3, 4, 6, 7, 10, 11, 13, 16 --> power 0.377 (0.448)
Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 15 --> power 0.287 (0.319)
Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 15 --> power 0.281 (0.288)
Optimal Design (normal approx.) when removing node/s 2, 6, 8, 9, 11, 15 with (normal) power 0.534 (0.612)

Exact Distribution. Optimal designs when reducing the number of points:

Complete Design power   --> 0.422
Removing node/s 7 --> power 0.442
Removing node/s 7, 10 --> power 0.47
Removing node/s 5, 7, 10 --> power 0.5
Removing node/s 3, 7, 10, 12 --> power 0.532
Removing node/s 3, 6, 11, 12, 14 --> power 0.573
Removing node/s 2, 7, 8, 9, 10, 15 --> power 0.612
Removing node/s 2, 6, 8, 9, 11, 13, 15 --> power 0.599
Removing node/s 1, 3, 5, 7, 10, 11, 12, 14 --> power 0.584
Removing node/s 1, 2, 4, 5, 8, 10, 11, 14, 15 --> power 0.471
Removing node/s 1, 2, 3, 4, 5, 8, 10, 11, 14, 15 --> power 0.448
Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 11, 12, 14 --> power 0.319
Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15 --> power 0.288
Optimal Design (exact) when removing node/s 2, 7, 8, 9, 10, 15 with power 0.612

Efficiency of the normal-optimal respect to the exact-optimal: 0.612 / 0.612 = 1
Computing time:
user system elapsed
1432.61   0.21 1433.03



GRID 4 x 4 ROOK'S RULE, ρ= 0.9 , RESULTS

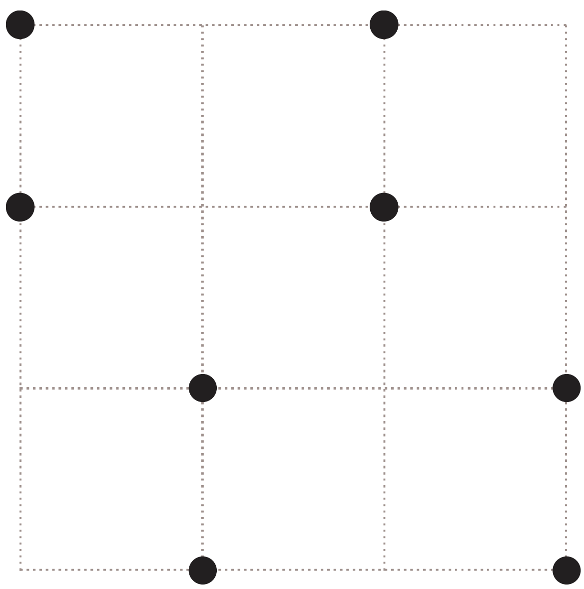
Normal Approximation. Optimal designs when reducing the number of points:

Complete Design power   --> 0.874 (0.868)
Removing node/s 7 --> power 0.863 (0.896)
Removing node/s 6, 10 --> power 0.891 (0.911)
Removing node/s 11, 12, 14 --> power 0.936 (0.959)
Removing node/s 2, 6, 9, 10 --> power 0.973 (0.96)
Removing node/s 2, 6, 7, 8, 9 --> power 0.988 (0.984)
Removing node/s 3, 5, 6, 7, 12, 16 --> power 0.998 (0.979)
Removing node/s 2, 4, 6, 8, 9, 11, 15 --> power 0.996 (0.996)
Removing node/s 1, 2, 7, 8, 9, 10, 11, 15 --> power 1 (0.996)
Removing node/s 1, 2, 3, 7, 10, 11, 12, 13, 14 --> power 0.841 (0.974)
Removing node/s 1, 2, 3, 4, 5, 8, 10, 11, 14, 15 --> power 0.993 (0.974)
Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 10, 14, 16 --> power 0.512 (0.848)
Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15 --> power 0.701 (0.842)
Optimal Design (normal approx.) when removing node/s 1, 2, 7, 8, 9, 10, 11, 15 with (normal) power 1 (0.996)

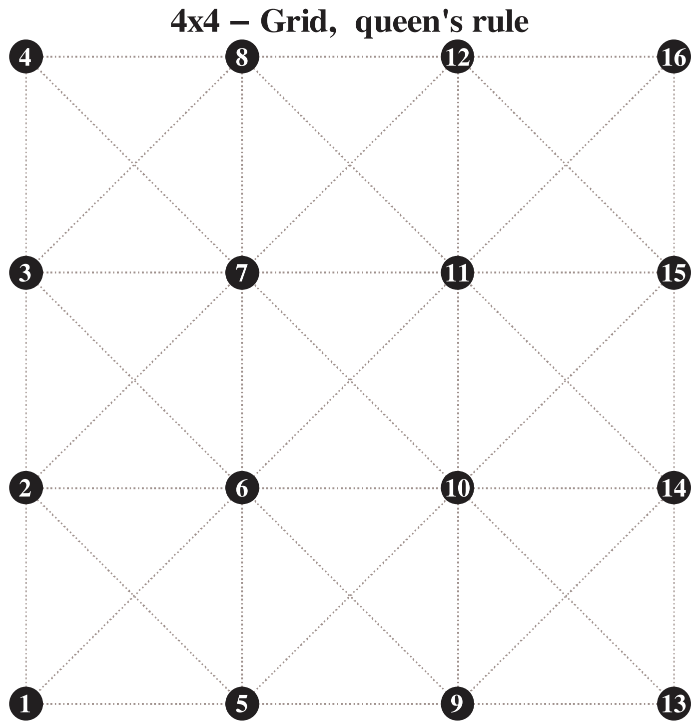
Exact Distribution. Optimal designs when reducing the number of points:

Complete Design power   --> 0.868
Removing node/s 7 --> power 0.896
Removing node/s 6, 11 --> power 0.928
Removing node/s 2, 7, 8 --> power 0.959
Removing node/s 2, 6, 9, 11 --> power 0.975
Removing node/s 6, 8, 9, 11, 15 --> power 0.988
Removing node/s 2, 6, 8, 9, 11, 15 --> power 0.996
Removing node/s 1, 3, 5, 7, 10, 12, 14 --> power 0.996
Removing node/s 1, 2, 7, 8, 9, 10, 15, 16 --> power 0.996
Removing node/s 1, 2, 3, 4, 5, 8, 10, 11, 15 --> power 0.974
Removing node/s 1, 2, 3, 4, 5, 8, 10, 11, 14, 15 --> power 0.974
Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 10, 14, 16 --> power 0.848
Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15 --> power 0.842
Optimal Design (exact) when removing node/s 1, 2, 7, 8, 9, 10, 15, 16 with power 0.996

Efficiency of the normal-optimal respect to the exact-optimal: 0.996 / 0.996 = 1
Computing time:
  user system elapsed
1944.86   0.22 1945.27



GRID 4x4
QUEEN'S RULE

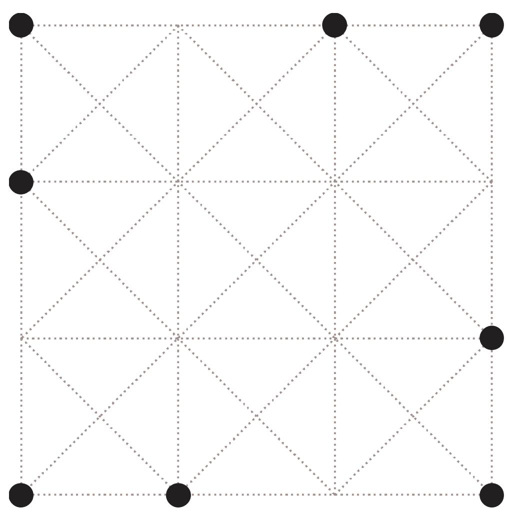


GRID 4 x 4 QUEEN'S RULE, ρ= 0.1 , RESULTS

Normal Approximation. Optimal designs when reducing the number of points:

Complete Design power --> 0.086 (0.078)
Removing node/s 11 --> power 0.088 (0.08)
Removing node/s 6, 7 --> power 0.091 (0.082)
Removing node/s 7, 10, 11 --> power 0.092 (0.083)
Removing node/s 6, 7, 11, 15 --> power 0.093 (0.085)
Removing node/s 6, 7, 8, 10, 14 --> power 0.095 (0.087)
Removing node/s 2, 6, 7, 8, 10, 14 --> power 0.096 (0.09)
Removing node/s 2, 6, 7, 8, 9, 10, 11 --> power 0.096 (0.09)
Removing node/s 3, 5, 6, 7, 10, 11, 12, 14 --> power 0.099 (0.093)
Removing node/s 1, 2, 5, 6, 7, 8, 10, 11, 14 --> power 0.093 (0.085)
Removing node/s 1, 2, 3, 5, 7, 10, 11, 12, 13, 14 --> power 0.091 (0.083)
Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 15 --> power 0.085 (0.075)
Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15 --> power 0.082 (0.072)
Optimal Design (normal approx.) when removing node/s 3, 5, 6, 7, 10, 11, 12, 14 with (normal) power 0.099 (0.093)

Exact Distribution. Optimal designs when reducing the number of points:



Complete Design power --> 0.078
Removing node/s 11 --> power 0.08
Removing node/s 6, 7 --> power 0.082
Removing node/s 2, 6, 10 --> power 0.084
Removing node/s 3, 5, 6, 7 --> power 0.087
Removing node/s 6, 7, 8, 11, 15 --> power 0.088
Removing node/s 3, 7, 10, 11, 12, 14 --> power 0.09
Removing node/s 5, 6, 7, 10, 11, 12, 14 --> power 0.09
Removing node/s 2, 6, 7, 8, 9, 10, 11, 15 --> power 0.093
Removing node/s 1, 5, 6, 7, 8, 9, 10, 11, 15 --> power 0.086
Removing node/s 1, 2, 3, 4, 7, 8, 9, 10, 11, 15 --> power 0.083
Removing node/s 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 15 --> power 0.075
Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 15 --> power 0.072
Optimal Design (exact) when removing node/s 2, 6, 7, 8, 9, 10, 11, 15 with power 0.093

Efficiency of the normal-optimal respect to the exact-optimal: 0.093 / 0.093 = 1

GRID 4 x 4 QUEEN'S RULE, ρ= 0.5 , RESULTS

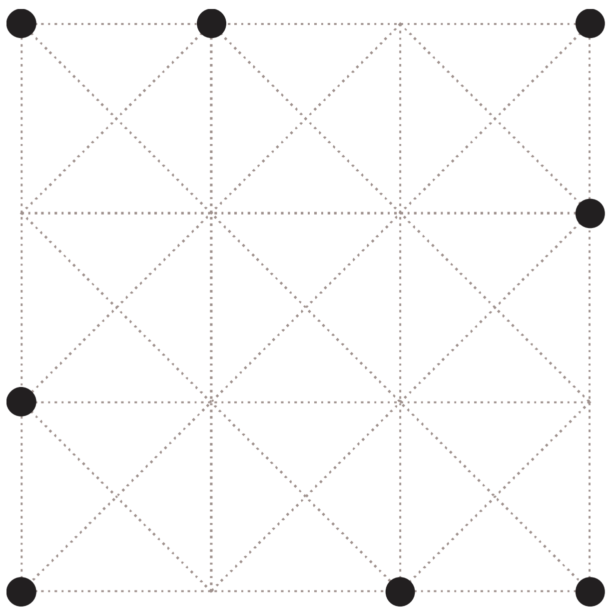
Normal Approximation. Optimal designs when reducing the number of points:

Complete Design power   --> 0.365 (0.317)
Removing node/s 6 --> power 0.389 (0.344)
Removing node/s 6, 10 --> power 0.416 (0.378)
Removing node/s 6, 7, 8 --> power 0.433 (0.403)
Removing node/s 5, 6, 10, 14 --> power 0.464 (0.448)
Removing node/s 3, 7, 10, 11, 12 --> power 0.477 (0.481)
Removing node/s 3, 7, 9, 10, 11, 12 --> power 0.495 (0.514)
Removing node/s 2, 6, 7, 8, 9, 10, 15 --> power 0.477 (0.526)
Removing node/s 3, 5, 6, 7, 10, 11, 12, 14 --> power 0.487 (0.584)
Removing node/s 1, 2, 5, 6, 7, 8, 11, 13, 15 --> power 0.428 (0.468)
Removing node/s 1, 2, 3, 4, 7, 8, 10, 11, 13, 14 --> power 0.378 (0.329)
Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 15 --> power 0.328 (0.318)
Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 15 --> power 0.281 (0.288)
Optimal Design (normal approx.) when removing node/s 3, 7, 9, 10, 11, 12 with (normal) power 0.495 (0.514)

Exact Distribution. Optimal designs when reducing the number of points:

Complete Design power   --> 0.317
Removing node/s 7 --> power 0.344
Removing node/s 6, 7 --> power 0.378
Removing node/s 3, 7, 11 --> power 0.403
Removing node/s 3, 5, 6, 7 --> power 0.448
Removing node/s 6, 7, 8, 11, 15 --> power 0.481
Removing node/s 7, 8, 9, 10, 11, 15 --> power 0.518
Removing node/s 2, 7, 8, 9, 10, 11, 15 --> power 0.526
Removing node/s 3, 5, 6, 7, 10, 11, 12, 14 --> power 0.584
Removing node/s 1, 2, 6, 7, 8, 9, 11, 13, 15 --> power 0.471
Removing node/s 1, 2, 3, 5, 7, 10, 11, 12, 13, 14 --> power 0.448
Removing node/s 1, 2, 3, 4, 5, 8, 10, 11, 12, 13, 14 --> power 0.319
Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15 --> power 0.288
Optimal Design (exact) when removing node/s 3, 5, 6, 7, 10, 11, 12, 14 with power 0.584

Efficiency of the normal-optimal respect to the exact-optimal: 0.514 / 0.584 = 0.88017
Computing time:
user system elapsed
2746.38   0.41 2746.94



```
 [1] GRID 4 x 4  QUEEN'S RULE, rho= 0.35 , RESULTS
[1]
[1] Normal Approximation. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power     -->  0.238 ( 0.201 )
[1] Removing node/s 10 --> power 0.252 ( 0.215 )
[1] Removing node/s 6, 7 --> power 0.268 ( 0.233 )
[1] Removing node/s 6, 7, 11 --> power 0.276 ( 0.245 )
[1] Removing node/s 7, 8, 11, 15 --> power 0.289 ( 0.27 )
[1] Removing node/s 2, 6, 7, 8, 10 --> power 0.297 ( 0.287 )
[1] Removing node/s 3, 7, 9, 10, 11, 15 --> power 0.307 ( 0.304 )
[1] Removing node/s 3, 5, 6, 7, 11, 12, 14 --> power 0.295 ( 0.309 )
[1] Removing node/s 2, 6, 7, 8, 9, 10, 11, 15 --> power 0.3 ( 0.337 )
[1] Removing node/s 1, 2, 5, 6, 7, 8, 9, 11, 15 --> power 0.271 ( 0.271 )
[1] Removing node/s 1, 2, 3, 4, 7, 8, 10, 11, 13, 14 --> power 0.25 ( 0.199 )
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 14 --> power 0.218 ( 0.192 )
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15 --> power 0.194 ( 0.174 )
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 --> power 0 ( 0 )
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 --> power 0 ( 0 )
[1] Optimal Design (normal approx.) when removing node/s 3, 7, 9, 10, 11, 15  with (normal) power 0.307 ( 0.304 )
[1]
[1] Exact Distribution. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power     -->  0.201
[1] Removing node/s 7 --> power 0.215
[1] Removing node/s 6, 7 --> power 0.233
[1] Removing node/s 2, 6, 10 --> power 0.246
[1] Removing node/s 10, 11, 12, 14 --> power 0.27
[1] Removing node/s 6, 10, 11, 12, 14 --> power 0.287
[1] Removing node/s 3, 5, 6, 7, 10, 14 --> power 0.306
[1] Removing node/s 2, 7, 8, 9, 10, 11, 15 --> power 0.309
[1] Removing node/s 3, 5, 6, 7, 10, 11, 12, 14 --> power 0.337
[1] Removing node/s 1, 2, 3, 7, 9, 10, 11, 12, 13 --> power 0.275
[1] Removing node/s 1, 2, 5, 6, 7, 8, 9, 11, 13, 15 --> power 0.257
[1] Removing node/s 1, 2, 3, 4, 5, 8, 10, 11, 12, 13, 14 --> power 0.194
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15 --> power 0.174
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 --> power 0
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 --> power 0
[1] Optimal Design (exact) when removing node/s 3, 5, 6, 7, 10, 11, 12, 14  with power 0.337
[1]
[1] Efficiency of the normal-optimal respect to the exact-optimal:  0.304 / 0.337 = 0.90278
[1]
[1] Date:
[1] 2010-01-15 09:15:04 Hora estándar romance
[1]
[1] Computing time:
   user  system elapsed
4034.26    2.72 4043.56
Hubo 50 o  más avisos (use warnings() para ver los primeros 50)
```

[1] GRID 4 x 4 QUEEN'S RULE, rho= 0.4 , RESULTS
[1]
[1] Normal Approximation. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power   --> 0.278 (0.236)
[1] Removing node/s 10 --> power 0.295 (0.254)
[1] Removing node/s 10, 11 --> power 0.314 (0.277)
[1] Removing node/s 10, 11, 12 --> power 0.325 (0.294)
[1] Removing node/s 5, 6, 10, 14 --> power 0.344 (0.324)
[1] Removing node/s 7, 9, 10, 11, 15 --> power 0.353 (0.346)
[1] Removing node/s 2, 6, 9, 10, 11, 12 --> power 0.365 (0.369)
[1] Removing node/s 3, 5, 6, 7, 11, 12, 14 --> power 0.35 (0.375)
[1] Removing node/s 2, 6, 7, 8, 9, 10, 11, 15 --> power 0.355 (0.414)
[1] Removing node/s 1, 2, 5, 6, 7, 8, 10, 14, 15 --> power 0.319 (0.33)
[1] Removing node/s 1, 2, 3, 4, 7, 8, 10, 11, 13, 14 --> power 0.29 (0.237)
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 15 --> power 0.252 (0.228)
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15 --> power 0.221 (0.207)
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 --> power 0 (0)
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 --> power 0 (0)
[1] Optimal Design (normal approx.) when removing node/s 2, 6, 9, 10, 11, 12 with (normal) power 0.365 (0.369)
[1]
[1] Exact Distribution. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power   --> 0.236
[1] Removing node/s 7 --> power 0.254
[1] Removing node/s 6, 7 --> power 0.277
[1] Removing node/s 3, 7, 11 --> power 0.294
[1] Removing node/s 3, 5, 6, 7 --> power 0.324
[1] Removing node/s 6, 7, 8, 11, 15 --> power 0.346
[1] Removing node/s 7, 8, 9, 10, 11, 15 --> power 0.371
[1] Removing node/s 2, 7, 8, 9, 10, 11, 15 --> power 0.375
[1] Removing node/s 3, 5, 6, 7, 10, 11, 12, 14 --> power 0.414
[1] Removing node/s 1, 2, 6, 7, 8, 9, 11, 13, 15 --> power 0.334
[1] Removing node/s 1, 2, 3, 4, 7, 8, 9, 10, 11, 15 --> power 0.313
[1] Removing node/s 1, 2, 3, 4, 5, 8, 10, 11, 12, 13, 14 --> power 0.231
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14 --> power 0.207
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 --> power 0
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 --> power 0
[1] Optimal Design (exact) when removing node/s 3, 5, 6, 7, 10, 11, 12, 14 with power 0.414
[1]
[1] Efficiency of the normal-optimal respect to the exact-optimal: 0.369 / 0.414 = 0.89218

```
[1] GRID 4 x 4  QUEEN'S RULE, rho= 0.45 , RESULTS
[1]
[1] Normal Approximation. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power     -->  0.321 ( 0.275 )
[1] Removing node/s 10 --> power 0.341 ( 0.297 )
[1] Removing node/s 10, 11 --> power 0.364 ( 0.325 )
[1] Removing node/s 10, 11, 12 --> power 0.378 ( 0.347 )
[1] Removing node/s 2, 6, 7, 8 --> power 0.402 ( 0.384 )
[1] Removing node/s 3, 5, 6, 7, 11 --> power 0.413 ( 0.411 )
[1] Removing node/s 3, 7, 9, 10, 11, 15 --> power 0.428 ( 0.44 )
[1] Removing node/s 2, 6, 7, 9, 10, 11, 15 --> power 0.411 ( 0.449 )
[1] Removing node/s 2, 6, 7, 8, 9, 10, 11, 15 --> power 0.417 ( 0.497 )
[1] Removing node/s 1, 3, 4, 7, 8, 9, 10, 11, 15 --> power 0.371 ( 0.396 )
[1] Removing node/s 1, 2, 3, 4, 5, 6, 10, 11, 15, 16 --> power 0.333 ( 0.281 )
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 11, 13, 15 --> power 0.289 ( 0.27 )
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14 --> power 0.251 ( 0.244 )
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 --> power 0 ( 0 )
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 --> power 0 ( 0 )
[1] Optimal Design (normal approx.) when removing node/s 3, 7, 9, 10, 11, 15  with (normal) power 0.428 ( 0.44 )
[1]
[1] Exact Distribution. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power     -->  0.275
[1] Removing node/s 10 --> power 0.297
[1] Removing node/s 6, 7 --> power 0.325
[1] Removing node/s 3, 7, 11 --> power 0.347
[1] Removing node/s 3, 5, 6, 7 --> power 0.384
[1] Removing node/s 6, 7, 8, 11, 15 --> power 0.411
[1] Removing node/s 7, 8, 9, 10, 11, 15 --> power 0.443
[1] Removing node/s 2, 7, 8, 9, 10, 11, 15 --> power 0.449
[1] Removing node/s 3, 5, 6, 7, 10, 11, 12, 14 --> power 0.497
[1] Removing node/s 1, 2, 3, 7, 9, 10, 11, 12, 13 --> power 0.4
[1] Removing node/s 1, 2, 3, 4, 6, 10, 11, 12, 13, 14 --> power 0.377
[1] Removing node/s 1, 2, 3, 4, 5, 8, 10, 11, 12, 13, 14 --> power 0.272
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15 --> power 0.244
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 --> power 0
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 --> power 0
[1] Optimal Design (exact) when removing node/s 3, 5, 6, 7, 10, 11, 12, 14  with power 0.497
[1]
[1] Efficiency of the normal-optimal respect to the exact-optimal:  0.44 / 0.497 = 0.88438
```

[1] GRID 4 x 4 QUEEN'S RULE, rho= 0.49 , RESULTS
[1]
[1] Normal Approximation. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power   --> 0.356 (0.308)
[1] Removing node/s 7 --> power 0.379 (0.335)
[1] Removing node/s 7, 11 --> power 0.406 (0.367)
[1] Removing node/s 10, 11, 12 --> power 0.422 (0.392)
[1] Removing node/s 10, 11, 12, 14 --> power 0.451 (0.435)
[1] Removing node/s 2, 6, 9, 10, 11 --> power 0.464 (0.467)
[1] Removing node/s 3, 7, 9, 10, 11, 15 --> power 0.482 (0.499)
[1] Removing node/s 3, 5, 6, 7, 11, 12, 14 --> power 0.463 (0.51)
[1] Removing node/s 2, 6, 7, 8, 9, 10, 11, 15 --> power 0.472 (0.567)
[1] Removing node/s 1, 2, 3, 7, 10, 11, 12, 13, 14 --> power 0.416 (0.453)
[1] Removing node/s 1, 2, 3, 4, 5, 6, 10, 11, 15, 16 --> power 0.369 (0.319)
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 15 --> power 0.32 (0.308)
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14 --> power 0.275 (0.279)
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 --> power 0 (0)
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 --> power 0 (0)
[1] Optimal Design (normal approx.) when removing node/s 3, 7, 9, 10, 11, 15 with (normal) power 0.482 (0.499)
[1]
[1] Exact Distribution. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power   --> 0.308
[1] Removing node/s 7 --> power 0.335
[1] Removing node/s 6, 7 --> power 0.367
[1] Removing node/s 3, 7, 11 --> power 0.392
[1] Removing node/s 3, 5, 6, 7 --> power 0.435
[1] Removing node/s 6, 7, 8, 11, 15 --> power 0.467
[1] Removing node/s 7, 8, 9, 10, 11, 15 --> power 0.503
[1] Removing node/s 2, 7, 8, 9, 10, 11, 15 --> power 0.51
[1] Removing node/s 3, 5, 6, 7, 10, 11, 12, 14 --> power 0.567
[1] Removing node/s 3, 4, 5, 6, 7, 11, 13, 14, 15 --> power 0.457
[1] Removing node/s 1, 2, 3, 4, 7, 8, 9, 10, 11, 15 --> power 0.433
[1] Removing node/s 1, 2, 3, 4, 5, 8, 10, 11, 12, 13, 14 --> power 0.309
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15 --> power 0.279
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 --> power 0
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 --> power 0
[1] Optimal Design (exact) when removing node/s 3, 5, 6, 7, 10, 11, 12, 14 with power 0.567
[1]
[1] Efficiency of the normal-optimal respect to the exact-optimal: 0.499 / 0.567 = 0.88069

[1] GRID 4 x 4 QUEEN'S RULE, rho= 0.51 , RESULTS
[1]
[1] Normal Approximation. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power   --> 0.375 (0.325)
[1] Removing node/s 6 --> power 0.399 (0.354)
[1] Removing node/s 6, 7 --> power 0.427 (0.389)
[1] Removing node/s 3, 7, 11 --> power 0.444 (0.415)
[1] Removing node/s 9, 10, 11, 15 --> power 0.476 (0.461)
[1] Removing node/s 6, 10, 11, 12, 14 --> power 0.491 (0.495)
[1] Removing node/s 3, 5, 6, 7, 11, 15 --> power 0.509 (0.53)
[1] Removing node/s 2, 6, 7, 8, 9, 10, 15 --> power 0.491 (0.542)
[1] Removing node/s 2, 6, 7, 8, 9, 10, 11, 15 --> power 0.502 (0.602)
[1] Removing node/s 2, 3, 6, 9, 10, 11, 12, 13, 14 --> power 0.44 (0.483)
[1] Removing node/s 1, 2, 3, 4, 7, 8, 10, 11, 13, 14 --> power 0.388 (0.339)
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 11, 13, 15 --> power 0.336 (0.328)
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15 --> power 0.288 (0.297)
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 --> power 0 (0)
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 --> power 0 (0)
[1] Optimal Design (normal approx.) when removing node/s 3, 5, 6, 7, 11, 15 with (normal) power 0.509 (0.53)
[1]
[1] Exact Distribution. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power   --> 0.325
[1] Removing node/s 10 --> power 0.354
[1] Removing node/s 6, 7 --> power 0.389
[1] Removing node/s 3, 7, 11 --> power 0.415
[1] Removing node/s 10, 11, 12, 14 --> power 0.461
[1] Removing node/s 6, 10, 11, 12, 14 --> power 0.495
[1] Removing node/s 7, 8, 9, 10, 11, 15 --> power 0.534
[1] Removing node/s 2, 7, 8, 9, 10, 11, 15 --> power 0.542
[1] Removing node/s 3, 5, 6, 7, 10, 11, 12, 14 --> power 0.602
[1] Removing node/s 3, 4, 5, 6, 7, 11, 13, 14, 15 --> power 0.486
[1] Removing node/s 1, 2, 3, 4, 6, 10, 11, 12, 13, 14 --> power 0.463
[1] Removing node/s 1, 2, 3, 4, 5, 8, 10, 11, 12, 13, 14 --> power 0.329
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14 --> power 0.297
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 --> power 0
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 --> power 0
[1] Optimal Design (exact) when removing node/s 3, 5, 6, 7, 10, 11, 12, 14 with power 0.602
[1]
[1] Efficiency of the normal-optimal respect to the exact-optimal: 0.53 / 0.602 = 0.87983

[1] GRID 4 x 4 QUEEN'S RULE, rho= 0.55 , RESULTS
[1]
[1] Normal Approximation. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power   --> 0.411 (0.362)
[1] Removing node/s 6 --> power 0.438 (0.394)
[1] Removing node/s 10, 11 --> power 0.47 (0.434)
[1] Removing node/s 2, 6, 10 --> power 0.49 (0.463)
[1] Removing node/s 10, 11, 12, 14 --> power 0.528 (0.515)
[1] Removing node/s 2, 6, 7, 8, 10 --> power 0.545 (0.553)
[1] Removing node/s 3, 7, 9, 10, 11, 12 --> power 0.567 (0.591)
[1] Removing node/s 3, 5, 6, 7, 10, 11, 12 --> power 0.549 (0.605)
[1] Removing node/s 3, 5, 6, 7, 10, 11, 12, 14 --> power 0.566 (0.672)
[1] Removing node/s 2, 3, 6, 9, 10, 11, 12, 13, 14 --> power 0.491 (0.545)
[1] Removing node/s 1, 2, 3, 4, 5, 6, 10, 11, 12, 14 --> power 0.43 (0.525)
[1] Removing node/s 1, 2, 3, 4, 5, 6, 10, 11, 12, 15, 16 --> power 0.371 (0.37)
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14 --> power 0.315 (0.338)
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 --> power 0 (0)
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 --> power 0 (0)
[1] Optimal Design (normal approx.) when removing node/s 3, 7, 9, 10, 11, 12 with (normal) power 0.567 (0.591)
[1]
[1] Exact Distribution. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power   --> 0.362
[1] Removing node/s 7 --> power 0.394
[1] Removing node/s 6, 7 --> power 0.434
[1] Removing node/s 3, 7, 11 --> power 0.463
[1] Removing node/s 3, 5, 6, 7 --> power 0.515
[1] Removing node/s 6, 7, 8, 11, 15 --> power 0.553
[1] Removing node/s 7, 8, 9, 10, 11, 15 --> power 0.595
[1] Removing node/s 2, 7, 8, 9, 10, 11, 15 --> power 0.605
[1] Removing node/s 3, 5, 6, 7, 10, 11, 12, 14 --> power 0.672
[1] Removing node/s 1, 2, 3, 7, 9, 10, 11, 12, 13 --> power 0.547
[1] Removing node/s 1, 2, 3, 4, 7, 8, 9, 10, 11, 15 --> power 0.525
[1] Removing node/s 1, 2, 3, 4, 5, 8, 10, 11, 12, 13, 14 --> power 0.371
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14 --> power 0.338
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 --> power 0
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 --> power 0
[1] Optimal Design (exact) when removing node/s 3, 5, 6, 7, 10, 11, 12, 14 with power 0.672
[1]
[1] Efficiency of the normal-optimal respect to the exact-optimal: 0.591 / 0.672 = 0.88025

[1] GRID 4 x 4 QUEEN'S RULE, rho= 0.6 , RESULTS
[1]
[1] Normal Approximation. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power   --> 0.458 (0.409)
[1] Removing node/s 11 --> power 0.489 (0.447)
[1] Removing node/s 10, 11 --> power 0.525 (0.492)
[1] Removing node/s 3, 7, 11 --> power 0.549 (0.525)
[1] Removing node/s 10, 11, 12, 14 --> power 0.594 (0.584)
[1] Removing node/s 2, 6, 7, 8, 10 --> power 0.615 (0.626)
[1] Removing node/s 5, 6, 10, 11, 12, 14 --> power 0.642 (0.672)
[1] Removing node/s 3, 5, 6, 7, 10, 11, 12 --> power 0.626 (0.683)
[1] Removing node/s 3, 5, 6, 7, 10, 11, 12, 14 --> power 0.655 (0.754)
[1] Removing node/s 1, 2, 3, 7, 10, 11, 12, 13, 14 --> power 0.56 (0.624)
[1] Removing node/s 1, 2, 5, 6, 7, 8, 9, 11, 13, 15 --> power 0.489 (0.605)
[1] Removing node/s 1, 2, 3, 4, 5, 6, 9, 10, 11, 15, 16 --> power 0.416 (0.429)
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 15 --> power 0.35 (0.395)
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 --> power 0 (0)
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 --> power 0 (0)
[1] Optimal Design (normal approx.) when removing node/s 3, 5, 6, 7, 10, 11, 12, 14 with (normal) power 0.655 (0.754)
[1]
[1] Exact Distribution. Optimal designs when reducing the number of points:
[1]
[1] Complete Design power   --> 0.409
[1] Removing node/s 7 --> power 0.447
[1] Removing node/s 6, 7 --> power 0.492
[1] Removing node/s 3, 7, 11 --> power 0.525
[1] Removing node/s 3, 5, 6, 7 --> power 0.584
[1] Removing node/s 6, 10, 11, 12, 14 --> power 0.626
[1] Removing node/s 7, 8, 9, 10, 11, 15 --> power 0.672
[1] Removing node/s 2, 7, 8, 9, 10, 11, 15 --> power 0.683
[1] Removing node/s 3, 5, 6, 7, 10, 11, 12, 14 --> power 0.754
[1] Removing node/s 1, 2, 3, 7, 9, 10, 11, 12, 13 --> power 0.624
[1] Removing node/s 1, 2, 3, 4, 6, 10, 11, 12, 13, 14 --> power 0.605
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 15 --> power 0.429
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 15 --> power 0.395
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 --> power 0
[1] Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 --> power 0
[1] Optimal Design (exact) when removing node/s 3, 5, 6, 7, 10, 11, 12, 14 with power 0.754
[1]
[1] Efficiency of the normal-optimal respect to the exact-optimal: 0.754 / 0.754 = 1

GRID 4 x 4 QUEEN'S RULE, ρ= 0.9 , RESULTS

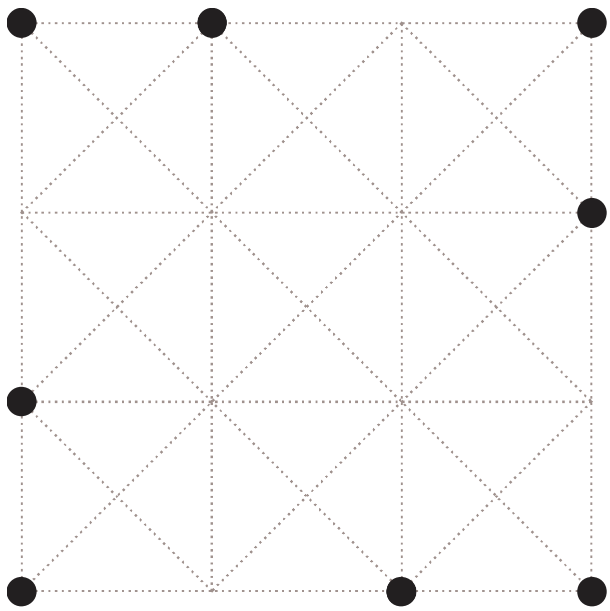
Normal Approximation. Optimal designs when reducing the number of points:

Complete Design power   --> 0.668 (0.712)
Removing node/s 6 --> power 0.729 (0.767)
Removing node/s 6, 10 --> power 0.797 (0.827)
Removing node/s 6, 7, 8 --> power 0.836 (0.869)
Removing node/s 2, 6, 9, 10 --> power 0.945 (0.936)
Removing node/s 3, 5, 6, 7, 11 --> power 0.97 (0.958)
Removing node/s 2, 6, 7, 8, 9, 10 --> power 0.996 (0.981)
Removing node/s 2, 6, 7, 8, 9, 11, 15 --> power 0.994 (0.984)
Removing node/s 2, 6, 7, 8, 9, 10, 11, 15 --> power 1 (0.996)
Removing node/s 1, 2, 3, 5, 6, 10, 11, 12, 14 --> power 0.999 (0.975)
Removing node/s 1, 2, 5, 6, 7, 8, 9, 11, 13, 15 --> power 0.993 (0.974)
Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 15 --> power 0.831 (0.854)
Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15 --> power 0.701 (0.842)
Optimal Design (normal approx.) when removing node/s 2, 6, 7, 8, 9, 10, 11, 15 with (normal) power 1 (0.996)

Exact Distribution. Optimal designs when reducing the number of points:

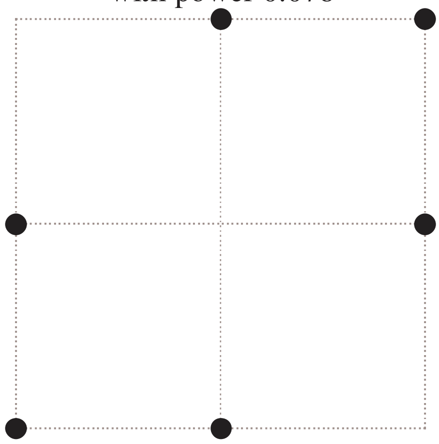
Complete Design power   --> 0.712
Removing node/s 6 --> power 0.767
Removing node/s 6, 10 --> power 0.827
Removing node/s 3, 7, 11 --> power 0.869
Removing node/s 3, 5, 6, 7 --> power 0.936
Removing node/s 2, 6, 9, 10, 11 --> power 0.958
Removing node/s 2, 6, 7, 8, 11, 15 --> power 0.981
Removing node/s 2, 7, 8, 9, 10, 11, 15 --> power 0.984
Removing node/s 3, 5, 6, 7, 10, 11, 12, 14 --> power 0.996
Removing node/s 1, 2, 5, 6, 7, 8, 10, 11, 14 --> power 0.975
Removing node/s 1, 2, 5, 6, 7, 8, 9, 11, 13, 15 --> power 0.974
Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 11, 13, 15 --> power 0.854
Removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15 --> power 0.842
Optimal Design (exact) when removing node/s 3, 5, 6, 7, 10, 11, 12, 14 with power 0.996

Efficiency of the normal-optimal respect to the exact-optimal: 0.996 / 0.996 = 1
Computing time:
user system elapsed
3420.54   0.52 3421.37

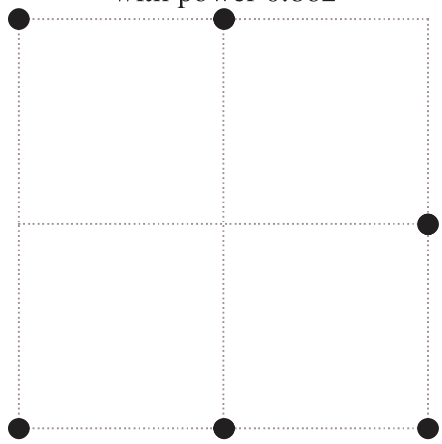


Some results for 3x3 and 4x4 grids:

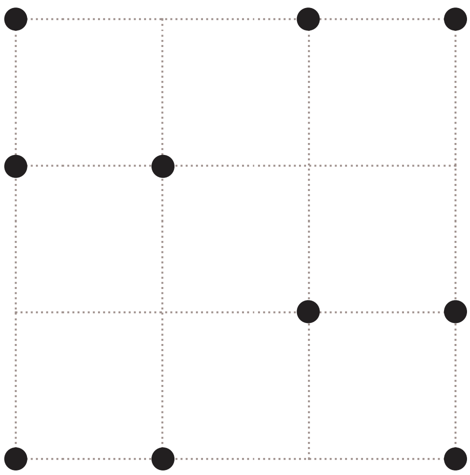
GRID 3X3, ROOK RULE, ρ= 0.1
Optimal Design (exact) when removing node/s 3, 5, 7
with power 0.078



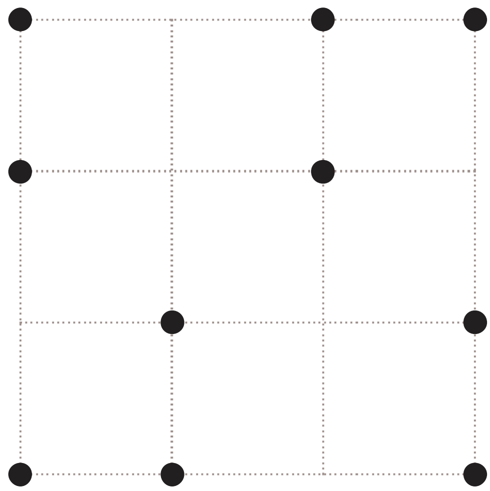
GRID 3X3, ROOK RULE, ρ= 0.9
Optimal Design (exact) when removing node/s 2, 5, 9
with power 0.862



GRID 4X4, ROOK RULE, ρ= 0.1
Optimal Design (exact) when removing node/s 2, 6, 8,
9, 11, 15 with power 0.097

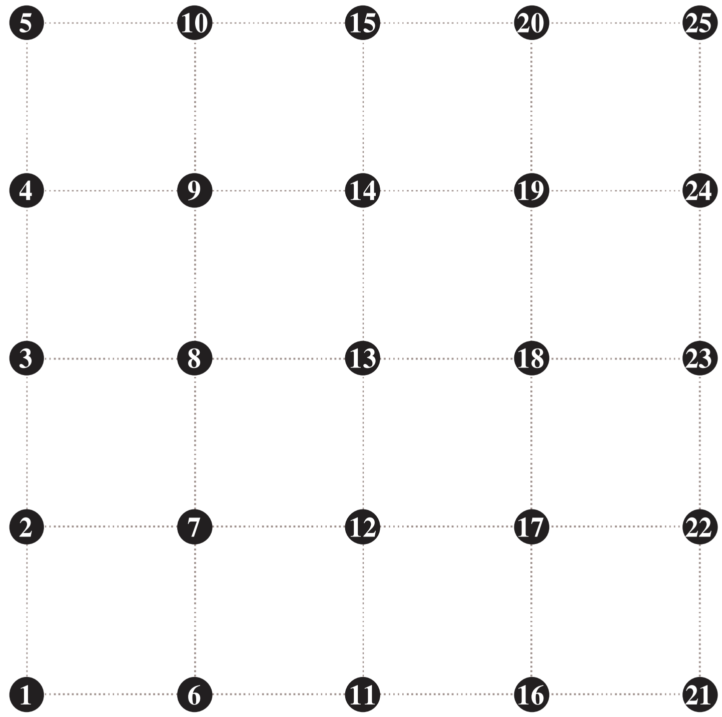


GRID 4X4, ROOK RULE, ρ= 0.5
Optimal Design (exact) when removing node/s 2, 7, 8,
9, 10, 15 with power 0.612



GRID 5x5, ROOK RULE

The nodes are numbered in the following way:



ρ= 0.5: RESULTS when removing <= 4

Normal Approximation. Optimal designs when reducing the number of points:

Complete Design power   --> 0.588
Removing node/s 17 --> power 0.597
Removing node/s 11, 17 --> power 0.613
Removing node/s 19, 20, 23 --> power 0.636
Removing node/s 16, 17, 19, 23 --> power 0.656
Optimal Design (normal approx.) when removing node/s 16, 17, 19, 23 with (normal) power 0.656

Exact Distribution. Optimal designs when reducing the number of points:

Complete Design power   --> 0.591
Removing node/s 9 --> power 0.604
Removing node/s 7, 11 --> power 0.62
Removing node/s 7, 11, 17 --> power 0.649
Removing node/s 16, 17, 19, 23 --> power 0.668
Optimal Design (exact) when removing node/s 16, 17, 19, 23 with power 0.668

Efficiency of the normal-optimal respect to the exact-optimal: 0.668 / 0.668 = 1

Computing time:

```
   user  system elapsed
1609.68    0.15 1612.95
```

GRID 5x5, ROOK RULE, ρ= 0.5 , RESULTS when removing nodes

Note: The last value in the normal approx. design (in brackets) is the exact power

GRID 5x5, ROOK RULE, ρ= 0.5 , RESULTS when removing 1 nodes

Optimal Design (normal approx.) when removing node/s 17 with (normal) power 0.597 (0.604)

Optimal Design (exact) when removing node/s 9 with power 0.604
Computing time:
user system elapsed
3.22 0.02 3.23

GRID 5x5, ROOK RULE, ρ= 0.5 , RESULTS when removing 2 nodes

Optimal Design (normal approx.) when removing node/s 11, 17 with (normal) power 0.613 (0.62)

Optimal Design (exact) when removing node/s 7, 11 with power 0.62
Computing time:
user system elapsed
37.05 0.00 37.11

GRID 5x5, ROOK RULE, ρ= 0.5 , RESULTS when removing 3 nodes

Optimal Design (normal approx.) when removing node/s 19, 20, 23 with (normal) power 0.636 (0.641)

Optimal Design (exact) when removing node/s 7, 11, 17 with power 0.649
Computing time:
user system elapsed
259.42 0.09 259.60

GRID 5x5, ROOK RULE, ρ= 0.5 , RESULTS when removing 4 nodes

Optimal Design (normal approx.) when removing node/s 16, 17, 19, 23 with (normal) power 0.656 (0.668)

Optimal Design (exact) when removing node/s 16, 17, 19, 23 with power 0.668
Computing time:
user system elapsed
1298.78 0.18 1299.00

GRID 5x5, ROOK RULE, ρ= 0.5 , RESULTS when removing 5 nodes

Optimal Design (normal approx.) when removing node/s 16, 17, 19, 20, 23 with (normal) power 0.674 (0.686)

Optimal Design (exact) when removing node/s 3, 7, 9, 15, 19 with power 0.688
Computing time:
user system elapsed
4850.76 0.63 4851.76

GRID 5x5, ROOK RULE, ρ= 0.5 , RESULTS when removing 6 nodes

Optimal Design (normal approx.) when removing node/s 7, 8, 11, 14, 17, 18 with (normal) power 0.69 (0.707)

Optimal Design (exact) when removing node/s 8, 9, 12, 15, 18, 19 with power 0.707
Computing time:

```
    user   system  elapsed 
13797.51     1.86 13800.43 
```

GRID 5x5, ROOK RULE, ρ= 0.5 , RESULTS when removing 7 nodes

Optimal Design (normal approx.) when removing node/s 11, 13, 15, 17, 19, 22, 24 with (normal) power 0.714 (0.736)

Optimal Design (exact) when removing node/s 3, 6, 7, 13, 16, 17, 23 with power 0.736
Computing time:

```
    user   system  elapsed 
31475.61     5.39 31488.51 
```

GRID 5x5, ROOK RULE, ρ= 0.5 , RESULTS when removing 8 nodes

Optimal Design (normal approx.) when removing node/s 3, 6, 7, 13, 14, 16, 17, 23 with (normal) power 0.721 (0.744)

Optimal Design (exact) when removing node/s 2, 4, 7, 9, 11, 13, 15, 22 with power 0.747
Computing time:

```
    user   system  elapsed 
59004.56    38.39 59063.91 
```

GRID 5x5, ROOK RULE, ρ= 0.5 , RESULTS when removing 9 nodes

Optimal Design (normal approx.) when removing node/s 3, 9, 10, 12, 13, 16, 19, 20, 23 with (normal) power 0.733 (0.763)

Optimal Design (exact) when removing node/s 2, 4, 7, 9, 11, 13, 15, 18, 22 with power 0.763
Computing time:

```
    user   system  elapsed 
86433.54    15.13 86462.30 
```

GRID 5x5, ROOK RULE, ρ= 0.5 , RESULTS when removing 10 nodes

Optimal Design (normal approx.) when removing node/s 1, 4, 7, 8, 9, 12, 15, 16, 18, 19 with (normal) power 0.738 (0.776)

Optimal Design (exact) when removing node/s 1, 4, 7, 8, 12, 14, 16, 17, 19, 23 with power 0.776
Computing time:

```
     user    system   elapsed 
108240.28     18.72 108293.07 
```

GRID 5x5, ROOK RULE, ρ= 0.5 , RESULTS when removing 11 nodes

Optimal Design (normal approx.) when removing node/s 3, 6, 7, 9, 12, 14, 15, 17, 18, 21, 24 with (normal) power 0.748 (0.787)

Optimal Design (exact) when removing node/s 2, 5, 7, 8, 9, 11, 14, 17, 18, 20, 22 with power 0.788
Computing time:
user system elapsed
110441.16 18.89 110488.25

GRID 5x5, ROOK RULE, ρ= 0.5 , RESULTS when removing 12 nodes

Optimal Design (normal approx.) when removing node/s 2, 5, 7, 8, 9, 12, 14, 16, 17, 19, 20, 23 with (normal) power 0.735 (0.782)

Optimal Design (exact) when removing node/s 4, 6, 7, 8, 9, 12, 15, 17, 18, 19, 21, 24 with power 0.782
Computing time:
user system elapsed
95929.20 14.62 95953.94

GRID 5x5, ROOK RULE, ρ= 0.5 , RESULTS when removing 13 nodes

Optimal Design (normal approx.) when removing node/s 2, 3, 7, 9, 10, 11, 12, 14, 18, 19, 21, 22, 25 with (normal) power 0.72 (0.776)

Optimal Design (exact) when removing node/s 1, 2, 5, 7, 8, 9, 12, 14, 16, 17, 19, 20, 23 with power 0.776
Computing time:
user system elapsed
70867.28 16.72 70923.67

GRID 5x5, ROOK RULE, ρ= 0.5 , RESULTS when removing 14 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 7, 9, 12, 14, 16, 17, 19, 20, 23 with (normal) power 0.627 (0.702)

Optimal Design (exact) when removing node/s 1, 3, 8, 9, 10, 11, 12, 13, 17, 19, 20, 22, 24, 25 with power 0.702
Computing time:
user system elapsed
45898.50 25.08 45937.41

GRID 5x5, ROOK RULE, ρ= 0.5 , RESULTS when removing 15 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 7, 9, 12, 14, 16, 17, 18, 20, 23, 25 with (normal) power 0.606 (0.693)

Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 8, 9, 11, 12, 14, 17, 19, 20, 22, 23 with power 0.693
Computing time:
user system elapsed
24369.67 4.52 24378.42

GRID 5x5, ROOK RULE, ρ= 0.5 , RESULTS when removing 16 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 7, 9, 12, 14, 15, 16, 17, 18, 21, 22, 23, 25 with (normal) power 0.511 (0.599)

Optimal Design (exact) when removing node/s 1, 2, 3, 6, 7, 8, 10, 11, 12, 14, 15, 17, 19, 22, 23, 24 with power 0.599
Computing time:
```
    user   system  elapsed 
11420.52     6.04 11432.77 
```

GRID 5x5, ROOK RULE, ρ= 0.5 , RESULTS when removing 17 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 16, 17, 18, 19, 20, 23 with (normal) power 0.487 (0.584)

Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 14, 16, 17, 19, 20, 23 with power 0.584
Computing time:
```
   user  system elapsed 
4675.05    0.81 4676.41 
```

GRID 5x5, ROOK RULE, ρ= 0.5 , RESULTS when removing 18 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 17, 19, 20, 22, 25 with (normal) power 0.398 (0.471)

Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 17, 19, 20, 22, 24, 25 with power 0.471
Computing time:
```
   user  system elapsed 
1603.42    7.09 1610.97 
```

GRID 5x5, ROOK RULE, ρ= 0.5 , RESULTS when removing 19 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 17, 18, 19, 22, 25 with (normal) power 0.377 (0.448)

Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 17, 18, 19, 22, 25 with power 0.448
Computing time:
```
   user  system elapsed 
 497.89    0.15  498.27 
```

GRID 5x5, ROOK RULE, ρ= 0.5 , RESULTS when removing 20 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 21, 24 with (normal) power 0.287 (0.319)

Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 21, 23 with power 0.319
Computing time:
```
   user  system elapsed 
 108.00    0.03  108.03 
```

GRID 5x5, ROOK RULE, ρ= 0.5 , RESULTS when removing 21 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 21, 22, 24 with (normal) power 0.281 (0.288)

Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23 with power 0.288
Computing time:

user	system	elapsed
93.55	0.00	93.58

GRID 5x5, ROOK RULE, ρ= 0.5 , RESULTS when removing 22 nodes

Optimal Design (normal approx.) when removing node/s with (normal) power 0 (0)

Optimal Design (exact) when removing node/s with power 0
Computing time:

user	system	elapsed
0.71	0.00	0.70

GRID 5x5, ROOK RULE, ρ= 0.5 , RESULTS when removing 23 nodes

Optimal Design (normal approx.) when removing node/s with (normal) power 0 (0)

Optimal Design (exact) when removing node/s with power 0
Computing time:

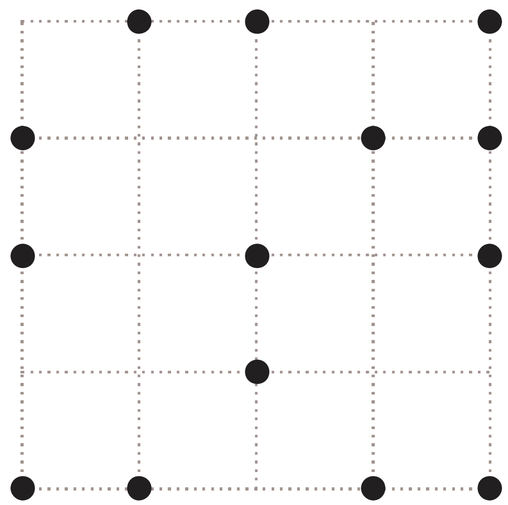
user	system	elapsed
0.25	0.00	0.25

****

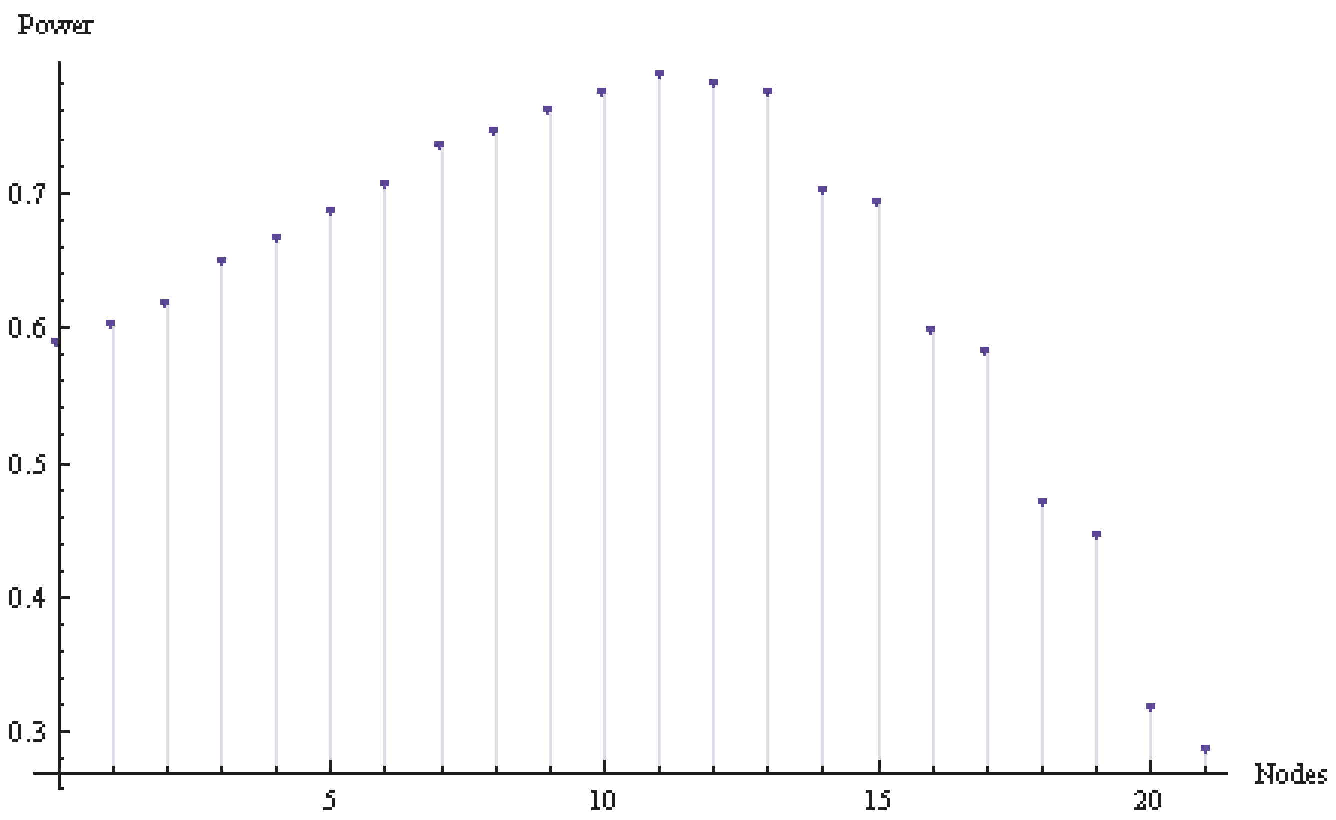
****************** **FINAL RESULTS** ******************

****

Thus the optimal design is a 14-point design, **removing node/s 2, 5, 7, 8, 9, 11, 14, 17, 18, 20, 22 with power 0.788,** that is



The graph for the maximum power depending on the number of nodes:



MATHEMATICA COMPUTATIONS FOR 5x5 GRID, P=0.5

Notes:

- In the function ‘fullEnumeration’ the parameter before ‘completeDes’ (that means the 25-point design) is the number of nodes that we want in the design. We begin in 25 and go down to 17. When computed the best 16-point design the Mathematica crashed.
- Output: the first value is the computing time (in seconds), the second is the power (normal) and the third the design. The power begin to be different that the one in spdep for normal approx. in the best 23-point design (0.613 –spdep- versus 0.616 –Mathematica-), but the difference is specially remarkable for the 21-point design (0.656 versus 0.734)

```
fullEnumeration[.01,{1},{x1,x2},5.76,.5,1,25,completeDes]//Timing
{0.062,{0.587904,{{-1.,-1.},{-1.,-0.5},{-1.,0.},{-1.,0.5},{-1.,1.},{-0.5,-1.},{-0.5,-0.5},{-0.5,0.},{-0.5,0.5},{-0.5,1.},{0.,-1.},{0.,-0.5},{0.,0.},{0.,0.5},{0.,1.},{0.5,-1.},{0.5,-0.5},{0.5,0.},{0.5,0.5},{0.5,1.},{1.,-1.},{1.,-0.5},{1.,0.},{1.,0.5},{1.,1.}}}}

fullEnumeration[.01,{1},{x1,x2},5.76,.5,1,24,completeDes]//Timing
{1.579,{0.597861,{{-1.,-1.},{-1.,-0.5},{-1.,0.},{-1.,0.5},{-1.,1.},{-0.5,-1.},{-0.5,0.},{-0.5,0.5},{-0.5,1.},{0.,-1.},{0.,-0.5},{0.,0.},{0.,0.5},{0.,1.},{0.5,-1.},{0.5,-0.5},{0.5,0.},{0.5,0.5},{0.5,1.},{1.,-1.},{1.,-0.5},{1.,0.},{1.,0.5},{1.,1.}}}}

fullEnumeration[.01,{1},{x1,x2},5.76,.5,1,23,completeDes]//Timing
{17.844,{0.615677,{{-1.,-1.},{-1.,-0.5},{-1.,0.},{-1.,0.5},{-1.,1.},{-0.5,-1.},{-0.5,-0.5},{-0.5,0.},{-0.5,0.5},{-0.5,1.},{0.,-1.},{0.,-0.5},{0.,0.},{0.,0.5},{0.,1.},{0.5,-1.},{0.5,-0.5},{0.5,0.},{0.5,1.},{1.,-1.},{1.,-0.5},{1.,0.5},{1.,1.}}}}

fullEnumeration[.01,{1},{x1,x2},5.76,.5,1,22,completeDes]//Timing
{124.828,{0.64127,{{-1.,-1.},{-1.,-0.5},{-1.,0.5},{-1.,1.},{-0.5,-1.},{-0.5,0.},{-0.5,1.},{0.,-1.},{0.,-0.5},{0.,0.},{0.,0.5},{0.,1.},{0.5,-1.},{0.5,-0.5},{0.5,0.},{0.5,0.5},{0.5,1.},{1.,-1.},{1.,-0.5},{1.,0.},{1.,0.5},{1.,1.}}}}

fullEnumeration[.01,{1},{x1,x2},5.76,.5,1,21,completeDes]//Timing
{624.625,{0.734148,{{-1.,-1.},{-1.,-0.5},{-1.,0.},{-1.,1.},{-0.5,-1.},{-0.5,-0.5},{-0.5,0.},{-0.5,1.},{0.,-1.},{0.,-0.5},{0.,0.},{0.,0.5},{0.5,-1.},{0.5,-0.5},{0.5,0.},{0.5,1.},{1.,-1.},{1.,-0.5},{1.,0.},{1.,0.5},{1.,1.}}}}

fullEnumeration[.01,{1},{x1,x2},5.76,.5,1,20,completeDes]//Timing
{2296.59,{0.742154,{{-1.,-1.},{-1.,-0.5},{-1.,0.},{-1.,1.},{-0.5,-1.},{-0.5,-0.5},{-0.5,0.},{-0.5,1.},{0.,-1.},{0.,0.},{0.,0.5},{0.5,-1.},{0.5,-0.5},{0.5,0.},{0.5,1.},{1.,-1.},{1.,-0.5},{1.,0.},{1.,0.5},{1.,1.}}}}

fullEnumeration[.01,{1},{x1,x2},5.76,.5,1,19,completeDes]//Timing
{6484.52,{0.7513,{{-1.,-1.},{-1.,-0.5},{-1.,0.5},{-1.,1.},{-0.5,0.},{-0.5,0.5},{-0.5,1.},{0.,-1.},{0.,-0.5},{0.,0.},{0.,0.5},{0.,1.},{0.5,-1.},{0.5,0.},{0.5,1.},{1.,-1.},{1.,-0.5},{1.,0.5},{1.,1.}}}}

fullEnumeration[.01,{1},{x1,x2},5.76,.5,1,18,completeDes]//Timing
{13799.4,{0.761346,{{-1.,-1.},{-1.,-0.5},{-1.,0.5},{-1.,1.},{-0.5,-1.},{-0.5,-0.5},{-0.5,0.},{0.,-1.},{0.,-0.5},{0.,0.},{0.,0.5},{0.,1.},{0.5,0.},{0.5,1.},{1.,-1.},{1.,-0.5},{1.,0.5},{1.,1.}}}}

fullEnumeration[.01,{1},{x1,x2},5.76,.5,1,17,completeDes]//Timing
{24387.7,{0.772628,{{-1.,-0.5},{-1.,0.},{-1.,1.},{-0.5,-1.},{-0.5,0.5},{-0.5,1.},{0.,-1.},{0.,0.},{0.,0.5},{0.5,-0.5},{0.5,0.},{0.5,1.},{1.,-1.},{1.,-0.5},{1.,0.},{1.,0.5},{1.,1.}}}}
```

Results after simmetrizing V in moransINormalApprox :

completeDes=Flatten[Table[{q,w},{q,-1,1,.5},{w,-1,1,.5}],1];fullEnumeration[.01,{1},{x1,x2},5.76,.5,1,25,completeDes]//Timing
{0.062,{0.58941,{{-1.,-1.},{-1.,-0.5},{-1.,0.},{-1.,0.5},{-1.,1.},{-0.5,-1.},{-0.5,-0.5},{-0.5,0.},{-0.5,0.5},{-0.5,1.},{0.,-1.},{0.,-0.5},{0.,0.},{0.,0.5},{0.,1.},{0.5,-1.},{0.5,-0.5},{0.5,0.},{0.5,0.5},{0.5,1.},{1.,-1.},{1.,-0.5},{1.,0.},{1.,0.5},{1.,1.}}}}

fullEnumeration[.01,{1},{x1,x2},5.76,.5,1,24,completeDes]//Timing
{1.516,{0.599187,{{-1.,-1.},{-1.,-0.5},{-1.,0.},{-1.,0.5},{-1.,1.},{-0.5,-1.},{-0.5,-0.5},{-0.5,0.},{-0.5,1.},{0.,-1.},{0.,-0.5},{0.,0.},{0.,0.5},{0.,1.},{0.5,-1.},{0.5,-0.5},{0.5,0.},{0.5,0.5},{0.5,1.},{1.,-1.},{1.,-0.5},{1.,0.},{1.,0.5},{1.,1.}}}}

fullEnumeration[.01,{1},{x1,x2},5.76,.5,1,23,completeDes]//Timing
{17.797,{0.625364,{{-1.,-1.},{-1.,-0.5},{-1.,0.},{-1.,0.5},{-1.,1.},{-0.5,-1.},{-0.5,-0.5},{-0.5,0.},{-0.5,0.5},{-0.5,1.},{0.,-1.},{0.,-0.5},{0.,0.},{0.,0.5},{0.,1.},{0.5,-1.},{0.5,0.},{0.5,0.5},{0.5,1.},{1.,-0.5},{1.,0.},{1.,0.5},{1.,1.}}}}

fullEnumeration[.01,{1},{x1,x2},5.76,.5,1,22,completeDes]//Timing
{126.828,{0.699363,{{-1.,-1.},{-1.,-0.5},{-1.,0.},{-1.,1.},{-0.5,-1.},{-0.5,-0.5},{-0.5,0.},{-0.5,1.},{0.,-1.},{0.,-0.5},{0.,0.},{0.,0.5},{0.5,-1.},{0.5,-0.5},{0.5,0.},{0.5,0.5},{0.5,1.},{1.,-1.},{1.,-0.5},{1.,0.},{1.,0.5},{1.,1.}}}}

fullEnumeration[.01,{1},{x1,x2},5.76,.5,1,21,completeDes]//Timing
{622.016,{0.711584,{{-1.,-1.},{-1.,-0.5},{-1.,0.},{-1.,0.5},{-1.,1.},{-0.5,-1.},{-0.5,-0.5},{-0.5,0.},{-0.5,0.5},{-0.5,1.},{0.,-0.5},{0.,0.},{0.,0.5},{0.,1.},{0.5,-1.},{0.5,0.},{0.5,1.},{1.,-1.},{1.,0.},{1.,0.5},{1.,1.}}}}

fullEnumeration[.01,{1},{x1,x2},5.76,.5,1,20,completeDes]//Timing
{2260.06,{0.735494,{{-1.,-1.},{-1.,-0.5},{-1.,0.},{-1.,0.5},{-1.,1.},{-0.5,-0.5},{-0.5,0.},{-0.5,0.5},{-0.5,1.},{0.,-1.},{0.,-0.5},{0.,0.},{0.,0.5},{0.,1.},{0.5,0.},{0.5,1.},{1.,-1.},{1.,-0.5},{1.,0.5},{1.,1.}}}}

fullEnumeration[.01,{1},{x1,x2},5.76,.5,1,19,completeDes]//Timing
{6179.58,{0.756751,{{-1.,-1.},{-1.,-0.5},{-1.,0.},{-1.,0.5},{-1.,1.},{-0.5,-1.},{-0.5,-0.5},{-0.5,0.},{-0.5,0.5},{-0.5,1.},{0.,-0.5},{0.,0.5},{0.,1.},{0.5,-1.},{0.5,0.},{0.5,1.},{1.,-1.},{1.,0.},{1.,1.}}}}

fullEnumeration[.01,{1},{x1,x2},5.76,.5,1,18,completeDes]//Timing
{13822.,{0.760332,{{-1.,-1.},{-1.,-0.5},{-1.,0.},{-1.,0.5},{-1.,1.},{-0.5,-1.},{-0.5,-0.5},{-0.5,0.},{-0.5,0.5},{0.,-0.5},{0.,0.5},{0.,1.},{0.5,-1.},{0.5,0.},{0.5,1.},{1.,-1.},{1.,0.},{1.,1.}}}}

fullEnumeration[.01,{1},{x1,x2},5.76,.5,1,17,completeDes]//Timing
{24476.2,{0.778041,{{-1.,-1.},{-1.,-0.5},{-1.,0.5},{-1.,1.},{-0.5,-1.},{-0.5,0.},{-0.5,1.},{0.,-0.5},{0.,0.},{0.,0.5},{0.5,-1.},{0.5,0.},{0.5,1.},{1.,-1.},{1.,-0.5},{1.,0.5},{1.,1.}}}}

GRID 5x5, ROOK RULE, ρ= 0.1 , RESULTS when removing <= 2

Note: The last value in the normal approx. design (in brackets) is the exact power

Normal Approximation. Optimal designs when reducing the number of points:

Complete Design power --> 0.099 (0.1)
Removing node/s 9 --> power 0.1 (0.101)
Removing node/s 7, 11 --> power 0.102 (0.102)
Optimal Design (normal approx.) when removing node/s 11, 17 with (normal) power 0.102 (0.102)

Exact Distribution. Optimal designs when reducing the number of points:

Complete Design power --> 0.1
Removing node/s 9 --> power 0.101
Removing node/s 3, 9 --> power 0.102
Optimal Design (exact) when removing node/s 3, 9 with power 0.102

Efficiency of the normal-optimal respect to the exact-optimal: 0.102 / 0.102 = 1

Computing time:

```
   user  system elapsed 
  33.46    0.00   33.49 
```

GRID 5x5, ROOK RULE, ρ= 0.1 , RESULTS when removing nodes

Note: The last value in the normal approx. design (in brackets) is the exact power

GRID 5x5, ROOK RULE, ρ= 0.1 , RESULTS when removing 1 nodes

Optimal Design (normal approx.) when removing node/s 9 with (normal) power 0.1 (0.101)

Optimal Design (exact) when removing node/s 9 with power 0.101
Computing time:

```
   user  system elapsed 
   2.64    0.00    2.64 
```

GRID 5x5, ROOK RULE, ρ= 0.1 , RESULTS when removing 2 nodes

Optimal Design (normal approx.) when removing node/s 7, 11 with (normal) power 0.102 (0.102)

Optimal Design (exact) when removing node/s 3, 9 with power 0.102
Computing time:

```
   user  system elapsed 
  30.42    0.09   30.53 
```

GRID 5x5, ROOK RULE, ρ= 0.1 , RESULTS when removing 3 nodes

Optimal Design (normal approx.) when removing node/s 3, 7, 9 with (normal) power 0.105 (0.105)

Optimal Design (exact) when removing node/s 7, 11, 17 with power 0.105
Computing time:

```
   user  system elapsed 
 219.30    0.12  237.29 
```

GRID 5x5, ROOK RULE, ρ= 0.1 , RESULTS when removing 4 nodes

Optimal Design (normal approx.) when removing node/s 7, 11, 17, 23 with (normal) power 0.107 (0.106)

Optimal Design (exact) when removing node/s 3, 7, 11, 17 with power 0.106
Computing time:

```
   user  system elapsed 
1105.18    0.14 1106.19 
```

GRID 5x5, ROOK RULE, ρ= 0.1 , RESULTS when removing 5 nodes

Optimal Design (normal approx.) when removing node/s 3, 7, 9, 11, 17 with (normal) power 0.109 (0.109)

Optimal Design (exact) when removing node/s 3, 7, 9, 15, 19 with power 0.109
Computing time:

```
   user  system elapsed 
4136.58    0.72 4141.98 
```

GRID 5x5, ROOK RULE, ρ= 0.1 , RESULTS when removing 6 nodes

Optimal Design (normal approx.) when removing node/s 3, 7, 9, 11, 15, 17 with (normal) power 0.111 (0.11)

Optimal Design (exact) when removing node/s 3, 7, 9, 11, 17, 23 with power 0.11
Computing time:

```
    user   system  elapsed 
12082.84     1.48 12097.58 
```

GRID 5x5, ROOK RULE, ρ= 0.1 , RESULTS when removing 7 nodes

Optimal Design (normal approx.) when removing node/s 3, 7, 9, 11, 15, 17, 19 with (normal) power 0.113 (0.112)

Optimal Design (exact) when removing node/s 3, 7, 9, 11, 15, 17, 19 with power 0.112
Computing time:

user	system	elapsed
27340.90	5.00	27359.77

GRID 5x5, ROOK RULE, ρ= 0.1 , RESULTS when removing 8 nodes

Optimal Design (normal approx.) when removing node/s 3, 7, 9, 11, 15, 17, 19, 23 with (normal) power 0.116 (0.115)

Optimal Design (exact) when removing node/s 3, 7, 9, 11, 15, 17, 19, 23 with power 0.115
Computing time:

user	system	elapsed
49910.38	6.72	49920.91

GRID 5x5, ROOK RULE, ρ= 0.1 , RESULTS when removing 9 nodes

Optimal Design (normal approx.) when removing node/s 3, 6, 7, 9, 15, 16, 17, 19, 23 with (normal) power 0.116 (0.115)

Optimal Design (exact) when removing node/s 3, 6, 7, 9, 15, 16, 17, 19, 23 with power 0.115
Computing time:

user	system	elapsed
75506.28	9.95	75522.12

GRID 5x5, ROOK RULE, ρ= 0.1 , RESULTS when removing 10 nodes

Optimal Design (normal approx.) when removing node/s 3, 7, 9, 10, 12, 14, 18, 19, 22, 25 with (normal) power 0.116 (0.114)

Optimal Design (exact) when removing node/s 3, 7, 9, 10, 12, 14, 18, 19, 22, 25 with power 0.114
Computing time:

user	system	elapsed
94547.20	15.29	94581.00

GRID 5x5, ROOK RULE, ρ= 0.1 , RESULTS when removing 11 nodes

Optimal Design (normal approx.) when removing node/s 1, 4, 7, 8, 12, 14, 16, 17, 19, 20, 23 with (normal) power 0.117 (0.115)

Optimal Design (exact) when removing node/s 2, 5, 8, 9, 12, 14, 16, 17, 19, 20, 23 with power 0.115
Computing time:

user	system	elapsed
98618.53	17.03	98671.36

GRID 5x5, ROOK RULE, ρ= 0.1 , RESULTS when removing 12 nodes

Optimal Design (normal approx.) when removing node/s 1, 4, 5, 7, 8, 12, 14, 15, 16, 17, 19, 23 with (normal) power 0.115 (0.113)

Optimal Design (exact) when removing node/s 2, 5, 7, 8, 9, 11, 14, 17, 18, 20, 22, 23 with power 0.113
Computing time:

```
    user   system  elapsed
85360.29    13.84 85394.55
```

GRID 5x5, ROOK RULE, ρ= 0.1 , RESULTS when removing 13 nodes

Optimal Design (normal approx.) when removing node/s 1, 4, 5, 7, 8, 12, 14, 15, 16, 17, 19, 23, 24 with (normal) power 0.114 (0.111)

Optimal Design (exact) when removing node/s 1, 4, 5, 7, 8, 12, 14, 15, 16, 17, 19, 20, 23 with power 0.111
Computing time:

```
    user   system  elapsed
63097.16    10.45 63120.34
```

GRID 5x5, ROOK RULE, ρ= 0.1 , RESULTS when removing 14 nodes

Optimal Design (normal approx.) when removing node/s 1, 4, 6, 7, 8, 11, 12, 14, 15, 17, 19, 22, 23, 24 with (normal) power 0.108 (0.104)

Optimal Design (exact) when removing node/s 1, 2, 5, 6, 8, 9, 11, 13, 16, 17, 18, 19, 20, 23 with power 0.104
Computing time:

```
    user   system  elapsed
39572.36     6.80 39622.02
```

GRID 5x5, ROOK RULE, ρ= 0.1 , RESULTS when removing 15 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 4, 5, 7, 9, 12, 13, 14, 16, 18, 20, 21, 23, 25 with (normal) power 0.107 (0.102)

Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 8, 11, 13, 14, 15, 17, 19, 22, 24 with power 0.102
Computing time:

```
    user   system  elapsed
20346.30     3.38 20351.86
```

GRID 5x5, ROOK RULE, ρ= 0.1 , RESULTS when removing 16 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 14, 16, 17, 18, 19, 20, 23 with (normal) power 0.1 (0.095)

Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 8, 9, 11, 13, 14, 17, 20, 22, 24, 25 with power 0.095
Computing time:

```
   user  system elapsed
9660.55    1.52 9663.10
```

GRID 5x5, ROOK RULE, ρ= 0.1 , RESULTS when removing 17 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 16, 17, 18, 19, 20, 23 with (normal) power 0.099 (0.093)

Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 11, 12, 14, 15, 17, 19, 22, 23, 24 with power 0.093
Computing time:
user system elapsed
4056.89 0.81 4058.19

GRID 5x5, ROOK RULE, ρ= 0.1 , RESULTS when removing 18 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 16, 18, 19, 20, 23 with (normal) power 0.092 (0.086)

Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 17, 19, 20, 22, 24, 25 with power 0.086
Computing time:
user system elapsed
1414.33 0.33 1414.78

GRID 5x5, ROOK RULE, ρ= 0.1 , RESULTS when removing 19 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 17, 19, 22, 23, 24 with (normal) power 0.091 (0.083)

Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 17, 18, 19, 22, 25 with power 0.083
Computing time:
user system elapsed
442.83 0.08 442.98

GRID 5x5, ROOK RULE, ρ= 0.1 , RESULTS when removing 20 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 23 with (normal) power 0.082 (0.075)

Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 24 with power 0.075
Computing time:
user system elapsed
98.10 0.03 98.12

GRID 5x5, ROOK RULE, ρ= 0.1 , RESULTS when removing 21 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 24 with (normal) power 0.082 (0.072)

Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 21, 22, 24 with power 0.072
Computing time:
user system elapsed
92.16 0.01 92.18

GRID 5x5, ROOK RULE, ρ= 0.1 , RESULTS when removing 22 nodes

Optimal Design (normal approx.) when removing node/s   with (normal) power 0 (0)

Optimal Design (exact) when removing node/s   with power 0
Computing time:

user	system	elapsed
0.7	0.0	0.7

GRID 5x5, ROOK RULE, ρ= 0.1 , RESULTS when removing 23 nodes

Optimal Design (normal approx.) when removing node/s   with (normal) power 0 (0)

Optimal Design (exact) when removing node/s   with power 0
Computing time:

user	system	elapsed
0.24	0.00	0.23

GRID 5x5, ROOK RULE, ρ= 0.9 , RESULTS when removing <= 2

Note: The last value in the normal approx. design (in brackets) is the exact power

Normal Approximation. Optimal designs when reducing the number of points:

Complete Design power --> 0.987 (0.975)
Removing node/s 17 --> power 0.99 (0.98)
Removing node/s 3, 8 --> power 0.991 (0.98)
Optimal Design (normal approx.) when removing node/s 3, 8 with (normal) power 0.991 (0.98)

Exact Distribution. Optimal designs when reducing the number of points:

Complete Design power --> 0.975
Removing node/s 9 --> power 0.98
Removing node/s 3, 7 --> power 0.985
Optimal Design (exact) when removing node/s 3, 7 with power 0.985

Efficiency of the normal-optimal respect to the exact-optimal: 0.98 / 0.985 = 0.99517

Computing time:

user	system	elapsed
61.80	0.10	61.95

GRID 5x5, ROOK RULE, ρ= 0.9 , RESULTS when removing nodes

Note: The last value in the normal approx. design (in brackets) is the exact power

GRID 5x5, ROOK RULE, ρ= 0.9 , RESULTS when removing 1 nodes

Optimal Design (normal approx.) when removing node/s 17 with (normal) power 0.99 (0.98)

Optimal Design (exact) when removing node/s 9 with power 0.98

```
 Computing time:
   user  system elapsed
   7.91    0.11   10.76
```

GRID 5x5, ROOK RULE, ρ= 0.9 , RESULTS when removing 2 nodes

Optimal Design (normal approx.) when removing node/s 3, 8 with (normal) power 0.991 (0.98)

Optimal Design (exact) when removing node/s 3, 7 with power 0.985

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```
Computing time:
   user  system elapsed
  56.83    1.00   58.29
H
```

GRID 5x5, ROOK RULE, ρ= 0.9 , RESULTS when removing 3 nodes

Optimal Design (normal approx.) when removing node/s 4, 9, 15 with (normal) power 0.998 (0.992)

Optimal Design (exact) when removing node/s 11, 17, 22 with power 0.992

```
 Computing time:
   user  system elapsed
 462.95    0.08  463.31
```

GRID 5x5, ROOK RULE, ρ= 0.9 , RESULTS when removing 4 nodes

Optimal Design (normal approx.) when removing node/s 2, 7, 11, 18 with (normal) power 0.999 (0.993)

Optimal Design (exact) when removing node/s 3, 7, 9, 10 with power 0.994

```
 Computing time:
   user  system elapsed
2031.56    0.39 2032.88
```

GRID 5x5, ROOK RULE, ρ= 0.9 , RESULTS when removing 5 nodes

Optimal Design (normal approx.) when removing node/s 2, 7, 11, 17, 22 with (normal) power 1 (0.997)

Optimal Design (exact) when removing node/s 2, 7, 11, 17, 22 with power 0.997

```
 Computing time:
   user  system elapsed
7624.38    0.91 7626.18
```

GRID 5x5, ROOK RULE, ρ= 0.9 , RESULTS when removing 6 nodes

Optimal Design (normal approx.) when removing node/s 2, 7, 11, 12, 17, 22 with (normal) power 1 (0.996)

Optimal Design (exact) when removing node/s 2, 4, 7, 9, 11, 13 with power 0.998
Computing time:

```
   user  system  elapsed
22013.29     4.62 22044.64
```

GRID 5x5, ROOK RULE, ρ= 0.9 , RESULTS when removing 7 nodes

Optimal Design (normal approx.) when removing node/s 2, 7, 11, 14, 15, 18, 23 with (normal) power 1 (0.997)

Optimal Design (exact) when removing node/s 2, 4, 7, 9, 11, 13, 15 with power 0.999
Computing time:

```
   user  system  elapsed
50646.14     7.28 50660.42
```

GRID 5x5, ROOK RULE, ρ= 0.9 , RESULTS when removing 8 nodes

Optimal Design (normal approx.) when removing node/s 2, 7, 11, 12, 18, 19, 20, 23 with (normal) power 1 (0.998)

Optimal Design (exact) when removing node/s 3, 7, 9, 11, 15, 17, 19, 23 with power 0.999
Computing time:

```
   user  system  elapsed
93495.94    11.99 93529.33
```

GRID 5x5, ROOK RULE, ρ= 0.9 , RESULTS when removing 9 nodes

Optimal Design (normal approx.) when removing node/s 2, 7, 9, 11, 12, 18, 19, 20, 23 with (normal) power 1 (0.999)

Optimal Design (exact) when removing node/s 2, 4, 7, 9, 11, 13, 15, 18, 22 with power 1
Computing time:

```
    user    system   elapsed
140055.65     18.33 140093.66
```

GRID 5x5, ROOK RULE, ρ= 0.9 , RESULTS when removing 10 nodes

Optimal Design (normal approx.) when removing node/s 1, 4, 7, 8, 9, 12, 15, 16, 17, 23 with (normal) power 1 (1)

Optimal Design (exact) when removing node/s 2, 5, 8, 9, 12, 14, 17, 19, 20, 23 with power 1
Computing time:

```
    user    system   elapsed
172316.58     21.06 172352.19
```

GRID 5x5, ROOK RULE, ρ= 0.9 , RESULTS when removing 11 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 5, 8, 9, 11, 12, 14, 19, 20, 23 with (normal) power 1 (1)

Optimal Design (exact) when removing node/s 1, 4, 7, 8, 9, 12, 15, 16, 17, 19, 23 with power 1
Computing time:
user system elapsed
176243.41 25.96 176313.71

GRID 5x5, ROOK RULE, ρ= 0.9 , RESULTS when removing 12 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 5, 8, 9, 11, 12, 14, 17, 19, 20, 23 with (normal) power 1 (1)

Optimal Design (exact) when removing node/s 1, 2, 5, 8, 9, 11, 12, 14, 17, 19, 20, 23 with power 1
Computing time:
user system elapsed
150610.66 19.96 150659.46

GRID 5x5, ROOK RULE, ρ= 0.9 , RESULTS when removing 13 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 7, 9, 12, 14, 15, 16, 17, 18, 23 with (normal) power 1 (0.999)

Optimal Design (exact) when removing node/s 1, 2, 5, 7, 8, 9, 12, 14, 16, 17, 19, 20, 23 with power 1
Computing time:
user system elapsed
108927.75 15.05 108953.75

GRID 5x5, ROOK RULE, ρ= 0.9 , RESULTS when removing 14 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 8, 11, 13, 15, 17, 19, 22, 24 with (normal) power 1 (0.999)

Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 7, 9, 12, 14, 16, 18, 20, 23, 25 with power 0.999
Computing time:
user system elapsed
67778.89 10.94 67839.31

GRID 5x5, ROOK RULE, ρ= 0.9 , RESULTS when removing 15 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 8, 10, 11, 13, 15, 17, 19, 22, 24 with (normal) power 1 (0.999)

Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 8, 10, 11, 13, 15, 17, 19, 22, 24 with power 0.999
Computing time:
user system elapsed
35666.37 6.77 35692.22

GRID 5x5, ROOK RULE, ρ= 0.9 , RESULTS when removing 16 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 14, 17, 18, 19, 22, 25 with (normal) power 0.996 (0.996)

Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 14, 17, 19, 20, 23 with power 0.996
Computing time:
user system elapsed
16369.19 3.44 16387.40

GRID 5x5, ROOK RULE, ρ= 0.9 , RESULTS when removing 17 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 14, 17, 18, 19, 22, 25 with (normal) power 1 (0.996)

Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 14, 17, 18, 19, 22, 25 with power 0.996
Computing time:
user system elapsed
6523.12 1.66 6528.11

GRID 5x5, ROOK RULE, ρ= 0.9 , RESULTS when removing 18 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 18, 19, 22, 25 with (normal) power 0.841 (0.974)

Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 17, 19, 20, 22, 25 with power 0.974
Computing time:
user system elapsed
2178.00 0.36 2178.50

GRID 5x5, ROOK RULE, ρ= 0.9 , RESULTS when removing 19 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 17, 19, 20, 22, 24, 25 with (normal) power 0.993 (0.974)

Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 17, 19, 20, 22, 24, 25 with power 0.974
Computing time:
user system elapsed
662.15 0.05 662.31

GRID 5x5, ROOK RULE, ρ= 0.9 , RESULTS when removing 20 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 21, 23, 25 with (normal) power 0.512 (0.848)

Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 21, 23, 25 with power 0.848
Computing time:
user system elapsed
146.09 0.00 146.09

GRID 5x5, ROOK RULE, ρ= 0.9 , RESULTS when removing 21 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 24 with (normal) power 0.701 (0.842)

Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 24 with power 0.842

Computing time:

user	system	elapsed
98.49	0.03	98.53

GRID 5x5, ROOK RULE, ρ= 0.9 , RESULTS when removing 22 nodes

Optimal Design (normal approx.) when removing node/s with (normal) power 0 (0)

Optimal Design (exact) when removing node/s with power 0

Computing time:

user	system	elapsed
0.69	0.00	0.69

GRID 5x5, ROOK RULE, ρ= 0.9 , RESULTS when removing 23 nodes

Optimal Design (normal approx.) when removing node/s with (normal) power 0 (0)

Optimal Design (exact) when removing node/s with power 0

Computing time:

user	system	elapsed
0.25	0.00	0.25

GRID 5x5, QUEEN RULE, ρ= 0.5 , RESULTS when removing <= 2

Note: The last value in the normal approx. design (in brackets) is the exact power

Normal Approximation. Optimal designs when reducing the number of points:

Complete Design power --> 0.516 (0.455)
Removing node/s 13 --> power 0.531 (0.468)
Removing node/s 13, 14 --> power 0.544 (0.483)
Optimal Design (normal approx.) when removing node/s 13, 14 with (normal) power 0.544 (0.483)

Exact Distribution. Optimal designs when reducing the number of points:

Complete Design power --> 0.455
Removing node/s 13 --> power 0.468
Removing node/s 12, 13 --> power 0.483
Optimal Design (exact) when removing node/s 12, 13 with power 0.483

Efficiency of the normal-optimal respect to the exact-optimal: 0.483 / 0.483 = 1

Computing time:

```
   user  system elapsed
  46.89    0.00   47.00
```

GRID 5x5, QUEEN RULE, ρ= 0.5 , RESULTS when removing 1 nodes

Optimal Design (normal approx.) when removing node/s 13 with (normal) power 0.531 (0.468)

Optimal Design (exact) when removing node/s 13 with power 0.468
Computing time:

```
   user  system elapsed 
   3.79    0.00    3.79 
```

GRID 5x5, QUEEN RULE, ρ= 0.5 , RESULTS when removing 2 nodes

Optimal Design (normal approx.) when removing node/s 13, 14 with (normal) power 0.544 (0.483)

Optimal Design (exact) when removing node/s 12, 13 with power 0.483
Computing time:

```
   user  system elapsed 
  42.84    0.00   42.89 
```

GRID 5x5, QUEEN RULE, ρ= 0.5 , RESULTS when removing 3 nodes

Optimal Design (normal approx.) when removing node/s 8, 13, 18 with (normal) power 0.559 (0.499)

Optimal Design (exact) when removing node/s 7, 8, 9 with power 0.502
Computing time:

```
   user  system elapsed 
 311.05    0.05  311.32 
```

GRID 5x5, QUEEN RULE, ρ= 0.5 , RESULTS when removing 4 nodes

Optimal Design (normal approx.) when removing node/s 14, 15, 19, 24 with (normal) power 0.584 (0.541)

Optimal Design (exact) when removing node/s 3, 6, 7, 8 with power 0.541
Computing time:

```
   user  system elapsed 
1595.43    0.15 1596.74 
```

GRID 5x5, QUEEN RULE, ρ= 0.5 , RESULTS when removing 5 nodes

Optimal Design (normal approx.) when removing node/s 3, 8, 11, 12, 13 with (normal) power 0.596 (0.53)

Optimal Design (exact) when removing node/s 7, 16, 17, 18, 23 with power 0.557
Computing time:

```
   user  system elapsed 
6197.11    2.54 6211.68 
```

GRID 5x5, QUEEN RULE, ρ= 0.5 , RESULTS when removing 6 nodes

Optimal Design (normal approx.) when removing node/s 16, 17, 18, 19, 20, 23 with (normal) power 0.62 (0.599)

Optimal Design (exact) when removing node/s 3, 6, 7, 8, 9, 10 with power 0.599
Computing time:

```
    user   system  elapsed 
18956.34     2.88 18962.69 
```

GRID 5x5, QUEEN RULE, ρ= 0.5 , RESULTS when removing 7 nodes

Optimal Design (normal approx.) when removing node/s 8, 16, 17, 18, 19, 20, 23 with (normal) power 0.628 (0.613)

Optimal Design (exact) when removing node/s 2, 7, 11, 12, 14, 17, 22 with power 0.613
Computing time:

```
   user   system  elapsed
46943.87     6.19 46955.80
```

GRID 5x5, QUEEN RULE, ρ= 0.5 , RESULTS when removing 8 nodes

Optimal Design (normal approx.) when removing node/s 6, 7, 8, 13, 16, 17, 18, 23 with (normal) power 0.647 (0.644)

Optimal Design (exact) when removing node/s 11, 12, 13, 14, 17, 19, 22, 24 with power 0.644
Computing time:

```
   user   system  elapsed
96254.58    14.70 96296.91
```

GRID 5x5, QUEEN RULE, ρ= 0.5 , RESULTS when removing 9 nodes

Optimal Design (normal approx.) when removing node/s 11, 12, 13, 14, 15, 17, 19, 22, 24 with (normal) power 0.674 (0.674)

Optimal Design (exact) when removing node/s 3, 6, 7, 8, 13, 16, 17, 18, 23 with power 0.674
Computing time:

```
    user    system   elapsed
163669.07     22.08 163704.49
```

GRID 5x5, QUEEN RULE, ρ= 0.5 , RESULTS when removing 10 nodes

Optimal Design (normal approx.) when removing node/s 2, 4, 7, 9, 11, 12, 13, 14, 15, 23 with (normal) power 0.677 (0.682)

Optimal Design (exact) when removing node/s 2, 4, 7, 9, 11, 12, 14, 15, 19, 24 with power 0.685
Computing time:

```
    user    system   elapsed
232467.87     34.34 232629.86
```

GRID 5x5, QUEEN RULE, ρ= 0.5 , RESULTS when removing 11 nodes

Optimal Design (normal approx.) when removing node/s 3, 6, 7, 8, 13, 14, 15, 16, 17, 18, 23 with (normal) power 0.695 (0.702)

Optimal Design (exact) when removing node/s 3, 6, 7, 8, 9, 10, 13, 18, 19, 20, 23 with power 0.713
Computing time:

```
    user    system   elapsed
278083.69     41.55 278240.26
```

GRID 5x5, QUEEN RULE, ρ= 0.5 , RESULTS when removing 12 nodes

Optimal Design (normal approx.) when removing node/s 2, 4, 7, 9, 12, 13, 14, 15, 17, 19, 22, 24 with (normal) power 0.695 (0.745)

Optimal Design (exact) when removing node/s 3, 6, 7, 8, 9, 10, 16, 17, 18, 19, 20, 23 with power 0.745
Computing time:
user system elapsed
280538.95 42.22 280615.06

GRID 5x5, QUEEN RULE, ρ= 0.5 , RESULTS when removing 13 nodes

Optimal Design (normal approx.) when removing node/s 2, 4, 7, 9, 11, 12, 13, 14, 15, 17, 19, 22, 24 with (normal) power 0.72 (0.776)

Optimal Design (exact) when removing node/s 2, 4, 7, 9, 11, 12, 13, 14, 15, 17, 19, 22, 24 with power 0.776
Computing time:
user system elapsed
238337.25 39.76 238461.68

GRID 5x5, QUEEN RULE, ρ= 0.5 , RESULTS when removing 14 nodes

Optimal Design (normal approx.) when removing node/s 1, 3, 4, 6, 8, 11, 12, 13, 14, 15, 17, 19, 22, 24 with (normal) power 0.642 (0.697)

Optimal Design (exact) when removing node/s 1, 3, 4, 7, 9, 11, 12, 13, 14, 15, 17, 19, 22, 24 with power 0.702
Computing time:
user system elapsed
169138.59 24.95 169219.69

GRID 5x5, QUEEN RULE, ρ= 0.5 , RESULTS when removing 15 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 7, 9, 12, 14, 15, 16, 17, 18, 19, 21, 24 with (normal) power 0.606 (0.693)

Optimal Design (exact) when removing node/s 1, 2, 3, 6, 7, 8, 9, 10, 13, 16, 17, 18, 19, 20, 23 with power 0.693
Computing time:
user system elapsed
100284.92 15.78 100330.41

GRID 5x5, QUEEN RULE, ρ= 0.5 , RESULTS when removing 16 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 6, 7, 9, 12, 14, 15, 17, 18, 19, 21, 22, 23 with (normal) power 0.534 (0.595)

Optimal Design (exact) when removing node/s 1, 2, 3, 6, 7, 9, 10, 12, 13, 14, 17, 19, 21, 22, 24, 25 with power 0.599
Computing time:
user system elapsed
49068.74 8.17 49085.85

GRID 5x5, QUEEN RULE, ρ= 0.5 , RESULTS when removing 17 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 16, 17, 18, 19, 20, 23 with (normal) power 0.487 (0.584)

Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 14, 16, 17, 18, 19, 20, 23 with power 0.584

Computing time:

```
    user   system  elapsed
19951.75     4.89 19963.16
```

GRID 5x5, QUEEN RULE, ρ= 0.5 , RESULTS when removing 18 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 11, 12, 13, 14, 15, 16, 19, 21, 22, 24 with (normal) power 0.428 (0.468)

Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 14, 15, 17, 18, 19, 21, 23, 24 with power 0.471

Computing time:

```
   user  system elapsed
6717.07    2.02 6724.47
```

GRID 5x5, QUEEN RULE, ρ= 0.5 , RESULTS when removing 19 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 18, 19, 21, 22, 23 with (normal) power 0.378 (0.329)

Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 17, 19, 22, 23, 24 with power 0.448

Computing time:

```
   user  system elapsed
1987.21    0.38 1987.86
```

GRID 5x5, QUEEN RULE, ρ= 0.5 , RESULTS when removing 20 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 23, 24 with (normal) power 0.328 (0.318)

Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 22, 24 with power 0.319

Computing time:

```
  user  system elapsed
 413.44    0.03  413.49
```

GRID 5x5, QUEEN RULE, ρ= 0.5 , RESULTS when removing 21 nodes

Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 21, 22, 24 with (normal) power 0.281 (0.288)

Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23 with power 0.288

Computing time:

```
  user  system elapsed
 272.08    0.03  272.11
```

GRID 5x5, QUEEN RULE, ρ= 0.5 , RESULTS when removing 22 nodes

Optimal Design (normal approx.) when removing node/s   with (normal) power 0 (0)

Optimal Design (exact) when removing node/s   with power 0
Computing time:
user system elapsed
1.67   0.00   1.67

GRID 5x5, QUEEN RULE, ρ= 0.5 , RESULTS when removing 23 nodes

Optimal Design (normal approx.) when removing node/s   with (normal) power 0 (0)

Optimal Design (exact) when removing node/s   with power 0
Computing time:
user system elapsed
0.42   0.00   0.42

{0.455, 0.468, 0.483, 0.502, 0.541, 0.557, 0.599, 0.613, 0.644, 0.674, 0.685, 0.713, 0.745, 0.776, 0.702, 0.693, 0.599, 0.584, 0.471, 0.448, 0.319, 0.288}

{0.597, 0.613, 0.636, 0.656, 0.674, 0.69, 0.714, 0.721, 0.733, 0.738, **0.748,** 0.735, 0.72, 0.627, 0.606, 0.511, 0.487, 0.398, 0.377, 0.287, 0.281}

```
[1] GRID 5x5, QUEEN RULE, rho= 0.1 , RESULTS when removing 1 nodes
[1]
[1] Optimal Design (normal approx.) when removing node/s 13  with (normal) power 0.1 ( 0.089 )
[1]
[1] Optimal Design (exact) when removing node/s 7  with power 0.089
 [1] Computing time:
   user  system elapsed 
   3.28    0.00    3.33 


[1] GRID 5x5, QUEEN RULE, rho= 0.1 , RESULTS when removing 2 nodes
[1]
[1] Optimal Design (normal approx.) when removing node/s 8, 13  with (normal) power 0.102 ( 0.09 )
[1]
[1] Optimal Design (exact) when removing node/s 7, 9  with power 0.09
 [1] Computing time:
   user  system elapsed 
  36.83    0.03   36.87 


[1] GRID 5x5, QUEEN RULE, rho= 0.1 , RESULTS when removing 3 nodes
[1]
[1] Optimal Design (normal approx.) when removing node/s 13, 14, 15  with (normal) power 0.104 ( 0.091 )
[1]
[1] Optimal Design (exact) when removing node/s 7, 8, 9  with power 0.092
 [1] Computing time:
   user  system elapsed 
 268.00    0.10  268.13 


[1] GRID 5x5, QUEEN RULE, rho= 0.1 , RESULTS when removing 4 nodes
[1]
[1] Optimal Design (normal approx.) when removing node/s 8, 13, 18, 23  with (normal) power 0.105 ( 0.092 )
[1]
[1] Optimal Design (exact) when removing node/s 3, 6, 7, 8  with power 0.094
 [1] Computing time:
   user  system elapsed 
1371.67    0.29 1372.08 


[1] GRID 5x5, QUEEN RULE, rho= 0.1 , RESULTS when removing 5 nodes
[1]
[1] Optimal Design (normal approx.) when removing node/s 3, 8, 11, 12, 13  with (normal) power 0.109 ( 0.093 )
[1]
[1] Optimal Design (exact) when removing node/s 3, 6, 7, 8, 17  with power 0.096
 [1] Computing time:
   user  system elapsed 
5318.43    0.53 5319.33 
```

```
[1] GRID 5x5, QUEEN RULE, rho= 0.1 , RESULTS when removing 6 nodes
[1]
[1] Optimal Design (normal approx.) when removing node/s 11, 12, 13, 14, 18, 23  with (normal) power 0.11
( 0.093 )
[1]
[1] Optimal Design (exact) when removing node/s 3, 6, 7, 8, 9, 10  with power 0.099
 [1] Computing time:
   user   system  elapsed
16242.03     2.14 16245.28
```

```
[1] GRID 5x5, QUEEN RULE, rho= 0.1 , RESULTS when removing 7 nodes
[1]
[1] Optimal Design (normal approx.) when removing node/s 3, 8, 11, 12, 13, 14, 15  with (normal) power 0.113
( 0.094 )
[1]
[1] Optimal Design (exact) when removing node/s 8, 9, 10, 13, 18, 19, 20  with power 0.1
 [1] Computing time:
   user   system  elapsed
40315.28     5.38 40323.54
```

```
[1] GRID 5x5, QUEEN RULE, rho= 0.1 , RESULTS when removing 8 nodes
[1]
[1] Optimal Design (normal approx.) when removing node/s 3, 8, 11, 12, 13, 14, 18, 23  with (normal) power 0.114
( 0.095 )
[1]
[1] Optimal Design (exact) when removing node/s 11, 12, 13, 14, 17, 19, 22, 24  with power 0.102
 [1] Computing time:
   user   system  elapsed
83218.15    11.52 83235.30
```

```
[1] GRID 5x5, QUEEN RULE, rho= 0.1 , RESULTS when removing 9 nodes
[1]
[1] Optimal Design (normal approx.) when removing node/s 3, 8, 11, 12, 13, 14, 15, 18, 23  with (normal) power
0.117 ( 0.097 )
[1]
[1] Optimal Design (exact) when removing node/s 3, 6, 7, 8, 13, 16, 17, 18, 23  with power 0.104
 [1] Computing time:
    user    system   elapsed
143164.18     23.02 143219.08
```

```
[1] GRID 5x5, QUEEN RULE, rho= 0.1 , RESULTS when removing 10 nodes
[1]
[1] Optimal Design (normal approx.) when removing node/s 1, 3, 8, 11, 12, 13, 14, 15, 18, 23  with (normal) power
0.116 ( 0.097 )
[1]
[1] Optimal Design (exact) when removing node/s 2, 7, 9, 11, 12, 14, 17, 19, 22, 24  with power 0.105
 [1] Computing time:
    user    system   elapsed
205469.49     29.53 205550.47
```

[1] GRID 5x5, QUEEN RULE, rho= 0.1 , RESULTS when removing 11 nodes
[1]
[1] Optimal Design (normal approx.) when removing node/s 1, 3, 4, 8, 11, 12, 13, 14, 15, 18, 23 with (normal) power 0.114 (0.096)
[1]
[1] Optimal Design (exact) when removing node/s 6, 7, 8, 9, 10, 13, 16, 17, 18, 19, 20 with power 0.107
[1] Computing time:
user system elapsed
247372.29 34.92 247451.31

[1] GRID 5x5, QUEEN RULE, rho= 0.1 , RESULTS when removing 12 nodes
[1]
[1] Optimal Design (normal approx.) when removing node/s 1, 3, 8, 9, 11, 12, 13, 14, 15, 16, 18, 23 with (normal) power 0.113 (0.096)
[1]
[1] Optimal Design (exact) when removing node/s 2, 4, 7, 9, 11, 12, 14, 15, 17, 19, 22, 24 with power 0.109
[1] Computing time:
user system elapsed
251502.14 35.92 251572.60

[1] GRID 5x5, QUEEN RULE, rho= 0.1 , RESULTS when removing 13 nodes
[1]
[1] Optimal Design (normal approx.) when removing node/s 3, 6, 7, 8, 9, 10, 13, 16, 17, 18, 19, 20, 23 with (normal) power 0.114 (0.111)
[1]
[1] Optimal Design (exact) when removing node/s 3, 6, 7, 8, 9, 10, 13, 16, 17, 18, 19, 20, 23 with power 0.111
[1] Computing time:
user system elapsed
210724.33 31.59 210789.90

[1] GRID 5x5, QUEEN RULE, rho= 0.1 , RESULTS when removing 14 nodes
[1]
[1] Optimal Design (normal approx.) when removing node/s 2, 3, 4, 5, 7, 9, 11, 12, 14, 17, 18, 19, 20, 23 with (normal) power 0.109 (0.103)
[1]
[1] Optimal Design (exact) when removing node/s 1, 3, 7, 8, 9, 10, 11, 12, 14, 15, 17, 19, 22, 24 with power 0.104
[1] Computing time:
user system elapsed
144512.6 19.7 144542.5

[1] GRID 5x5, QUEEN RULE, rho= 0.1 , RESULTS when removing 15 nodes
[1]
[1] Optimal Design (normal approx.) when removing node/s 2, 3, 4, 7, 9, 11, 12, 14, 15, 17, 18, 19, 21, 23, 24 with (normal) power 0.107 (0.102)
[1]
[1] Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 8, 11, 12, 13, 14, 15, 17, 19, 22, 24 with power 0.102
[1] Computing time:
user system elapsed
85914.89 13.61 85934.89

[1] GRID 5x5, QUEEN RULE, rho= 0.1 , RESULTS when removing 16 nodes
[1]
[1] Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 8, 11, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23 with (normal) power 0.102 (0.087)
[1]
[1] Optimal Design (exact) when removing node/s 1, 2, 4, 6, 7, 9, 11, 12, 13, 14, 15, 17, 18, 22, 24, 25 with power 0.095
[1] Computing time:
user system elapsed
42835.55 6.60 42847.74

[1] GRID 5x5, QUEEN RULE, rho= 0.1 , RESULTS when removing 17 nodes
[1]
[1] Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 16, 17, 18, 19, 20, 23 with (normal) power 0.099 (0.093)
[1]
[1] Optimal Design (exact) when removing node/s 1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 14, 15, 17, 19, 22, 23, 24 with power 0.093
[1] Computing time:
user system elapsed
17751.09 3.18 17755.53

[1] GRID 5x5, QUEEN RULE, rho= 0.1 , RESULTS when removing 18 nodes
[1]
[1] Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 16, 18, 19, 20, 23 with (normal) power 0.093 (0.085)
[1]
[1] Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 14, 15, 17, 18, 19, 21, 23, 24 with power 0.086
[1] Computing time:
user system elapsed
6042.21 0.91 6043.45

[1] GRID 5x5, QUEEN RULE, rho= 0.1 , RESULTS when removing 19 nodes
[1]
[1] Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 17, 19, 22, 23, 24 with (normal) power 0.091 (0.083)
[1]
[1] Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 17, 19, 22, 23, 24 with power 0.083
[1] Computing time:
user system elapsed
1819.21 0.28 1819.56

[1] GRID 5x5, QUEEN RULE, rho= 0.1 , RESULTS when removing 20 nodes
[1]
[1] Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 21, 24 with (normal) power 0.085 (0.075)
[1]
[1] Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 24 with power 0.075
[1] Computing time:

```
  user  system elapsed
387.29    0.10  387.44
```

[1] GRID 5x5, QUEEN RULE, rho= 0.1 , RESULTS when removing 21 nodes
[1]
[1] Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 24 with (normal) power 0.082 (0.072)
[1]
[1] Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 21, 22, 24 with power 0.072
[1] Computing time:

```
  user  system elapsed
272.16    0.01  272.19
```

[1] GRID 5x5, QUEEN RULE, rho= 0.1 , RESULTS when removing 22 nodes
[1]
[1] Optimal Design (normal approx.) when removing node/s with (normal) power 0 (0)
[1]
[1] Optimal Design (exact) when removing node/s with power 0
[1] Computing time:

```
  user  system elapsed
  1.69    0.00    1.70
```

[1] GRID 5x5, QUEEN RULE, rho= 0.1 , RESULTS when removing 23 nodes
[1]
[1] Optimal Design (normal approx.) when removing node/s with (normal) power 0 (0)
[1]
[1] Optimal Design (exact) when removing node/s with power 0
[1] Computing time:

```
  user  system elapsed
  0.44    0.00    0.44
```

[1] GRID 5x5, QUEEN RULE, rho= 0.9 , RESULTS when removing 1 nodes
Optimal Design (normal approx.) when removing node/s 13 with (normal) power 0.931 (0.92)
[1] Optimal Design (exact) when removing node/s 13 with power 0.92

[1] GRID 5x5, QUEEN RULE, rho= 0.9 , RESULTS when removing 2 nodes
[1] Optimal Design (normal approx.) when removing node/s 8, 13 with (normal) power 0.947 (0.935)
[1] Optimal Design (exact) when removing node/s 13, 18 with power 0.935

[1] GRID 5x5, QUEEN RULE, rho= 0.9 , RESULTS when removing 3 nodes
[1] Optimal Design (normal approx.) when removing node/s 11, 12, 13 with (normal) power 0.967 (0.947)
[1] Optimal Design (exact) when removing node/s 12, 13, 14 with power 0.949

[1] GRID 5x5, QUEEN RULE, rho= 0.9 , RESULTS when removing 4 nodes
[1] Optimal Design (normal approx.) when removing node/s 3, 6, 7, 8 with (normal) power 0.985 (0.974)
[1] Optimal Design (exact) when removing node/s 2, 7, 11, 12 with power 0.974

[1] GRID 5x5, QUEEN RULE, rho= 0.9 , RESULTS when removing 5 nodes
[1] Optimal Design (normal approx.) when removing node/s 11, 12, 13, 18, 23 with (normal) power 0.996 (0.975)
[1] Optimal Design (exact) when removing node/s 2, 7, 11, 12, 14 with power 0.979

[1] GRID 5x5, QUEEN RULE, rho= 0.9 , RESULTS when removing 6 nodes
[1] Optimal Design (normal approx.) when removing node/s 2, 7, 11, 12, 17, 22 with (normal) power 0.999 (0.991)
[1] Optimal Design (exact) when removing node/s 2, 7, 11, 12, 17, 22 with power 0.991

[1] GRID 5x5, QUEEN RULE, rho= 0.9 , RESULTS when removing 7 nodes
[1] Optimal Design (normal approx.) when removing node/s 3, 8, 11, 12, 13, 14, 15 with (normal) power 1 (0.989)
[1] Optimal Design (exact) when removing node/s 2, 7, 11, 12, 14, 17, 22 with power 0.993

[1] GRID 5x5, QUEEN RULE, rho= 0.9 , RESULTS when removing 8 nodes
[1] Optimal Design (normal approx.) when removing node/s 3, 8, 11, 12, 13, 14, 15, 18 with (normal) power 1 (0.991)
[1] Optimal Design (exact) when removing node/s 2, 4, 7, 9, 11, 12, 13, 14 with power 0.996

[1] GRID 5x5, QUEEN RULE, rho= 0.9 , RESULTS when removing 9 nodes
[1] Optimal Design (normal approx.) when removing node/s 3, 8, 11, 12, 13, 14, 15, 18, 23 with (normal) power 1 (0.996)
[1] Optimal Design (exact) when removing node/s 2, 4, 7, 9, 11, 12, 13, 14, 15 with power 0.998

[1] GRID 5x5, QUEEN RULE, rho= 0.9 , RESULTS when removing 10 nodes
[1] Optimal Design (normal approx.) when removing node/s 2, 4, 7, 9, 11, 12, 13, 14, 15, 18 with (normal) power 1 (0.999)
[1] Optimal Design (exact) when removing node/s 2, 4, 7, 9, 11, 12, 13, 14, 15, 18 with power 0.999

[1] GRID 5x5, QUEEN RULE, rho= 0.9 , RESULTS when removing 11 nodes
[1] Optimal Design (normal approx.) when removing node/s 2, 4, 7, 9, 11, 12, 13, 14, 15, 17, 22 with (normal) power 1 (0.999)
[1] Optimal Design (exact) when removing node/s 2, 4, 7, 9, 11, 12, 13, 14, 15, 17, 22 with power 0.999

[1] GRID 5x5, QUEEN RULE, rho= 0.9 , RESULTS when removing 12 nodes
[1] Optimal Design (normal approx.) when removing node/s 1, 3, 8, 9, 10, 11, 12, 13, 18, 19, 20, 23 with (normal) power 1 (0.999)
[1] Optimal Design (exact) when removing node/s 2, 4, 7, 9, 11, 12, 13, 14, 17, 19, 22, 24 with power 1

GRID 5x5, QUEEN RULE, rho= 0.9 , RESULTS when removing 13 nodes
[1] Optimal Design (normal approx.) when removing node/s 1, 2, 3, 8, 9, 10, 11, 12, 13, 18, 19, 20, 23 with (normal) power 1 (0.999)
[1] Optimal Design (exact) when removing node/s 2, 4, 7, 9, 11, 12, 13, 14, 15, 17, 19, 22, 24 with power 1

[1] GRID 5x5, QUEEN RULE, rho= 0.9 , RESULTS when removing 14 nodes
[1] Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 7, 9, 12, 14, 15, 16, 17, 18, 19, 23 with (normal) power 1 (0.999)
[1] Optimal Design (exact) when removing node/s 1, 2, 4, 7, 9, 12, 13, 14, 15, 16, 17, 18, 23, 24 with power 0.999

[1] GRID 5x5, QUEEN RULE, rho= 0.9 , RESULTS when removing 15 nodes
[1] Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 7, 9, 12, 14, 16, 17, 18, 19, 20, 23 with (normal) power 1 (0.999)
[1] Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 7, 9, 12, 14, 16, 17, 18, 19, 20, 23 with power 0.999

[1] GRID 5x5, QUEEN RULE, rho= 0.9 , RESULTS when removing 16 nodes
[1] Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 13, 16, 17, 18, 19, 20, 23 with (normal) power 1 (0.996)
[1] Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 13, 16, 17, 18, 19, 20, 23 with power 0.996

[1] GRID 5x5, QUEEN RULE, rho= 0.9 , RESULTS when removing 17 nodes
[1] Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 16, 17, 18, 19, 20, 23 with (normal) power 1 (0.996)
[1] Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 16, 17, 18, 19, 20, 23 with power 0.996

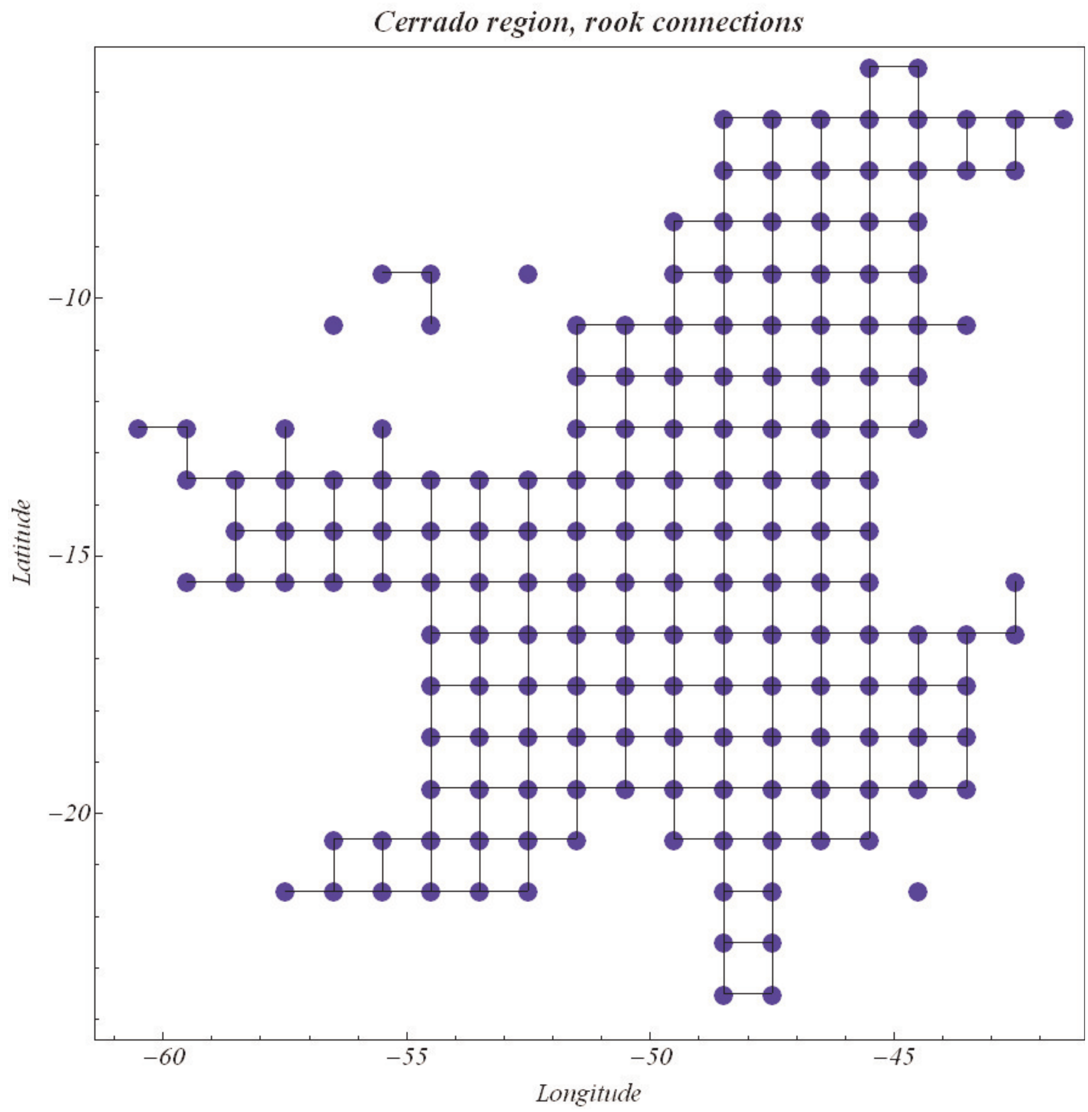
[1] GRID 5x5, QUEEN RULE, rho= 0.9 , RESULTS when removing 18 nodes
[1] Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 16, 17, 18, 19, 20, 23 with (normal) power 0.999 (0.975)
[1] Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 17, 19, 20, 22, 25 with power 0.975

[1] GRID 5x5, QUEEN RULE, rho= 0.9 , RESULTS when removing 19 nodes
[1] Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 17, 19, 21, 22, 24 with (normal) power 0.993 (0.974)
[1] Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 17, 19, 21, 22, 24 with power 0.974

[1] GRID 5x5, QUEEN RULE, rho= 0.9 , RESULTS when removing 20 nodes
[1] Optimal Design (normal approx.) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 21, 24 with (normal) power 0.831 (0.854)
[1] Optimal Design (exact) when removing node/s 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 23, 24 with power 0.854

[1] GRID 5x5, QUEEN RULE, rho= 0.9 , RESULTS when removing 23 nodes
[1] Optimal Design (normal approx.) when removing node/s with (normal) power NaN (0)
[1] Optimal Design (exact) when removing node/s with power NaN

CERRADO (BRAZIL): SIMPLE SEARCH ALGORITHM, RHO=0.5 ROOK'S RULE REMOVING PREVIOUSLY ISOLATED NODES {26, 33, 177}



Number of remaining nodes : 178, power: 0.999913607217003

* Best 177 -point design when removing node 118 with power 0.99992309332703
* Best 176 -point design when removing node 7 with power 0.999929702703025
* Best 175 -point design when removing node 6 with power 0.999937240557082
* Best 174 -point design when removing node 127 with power 0.999942185869714
* Best 173 -point design when removing node 156 with power 0.999946502100758
* Best 172 -point design when removing node 149 with power 0.999950356779081
* Best 171 -point design when removing node 138 with power 0.99995475753595
* Best 170 -point design when removing node 157 with power 0.99995786100893
* Best 169 -point design when removing node 147 with power 0.999962486447082
* Best 168 -point design when removing node 167 with power 0.999965512396257
* Best 167 -point design when removing node 166 with power 0.99996880077724
* Best 166 -point design when removing node 155 with power 0.999972722030138
* Best 165 -point design when removing node 17 with power 0.99997451441064
* Best 164 -point design when removing node 10 with power 0.999977237361143
* Best 163 -point design when removing node 4 with power 0.999979101472302
* Best 162 -point design when removing node 16 with power 0.999980759830912
* Best 161 -point design when removing node 10 with power 0.999983015903085
* Best 160 -point design when removing node 28 with power 0.999984205035424
* Best 159 -point design when removing node 20 with power 0.999986104794175

* Best 158 -point design when removing node 34 with power 0.99998740971057
* Best 157 -point design when removing node 28 with power 0.99998839995429
* Best 156 -point design when removing node 21 with power 0.99998970466245
* Best 155 -point design when removing node 28 with power 0.999990510536137
* Best 154 -point design when removing node 22 with power 0.999991434979395
* Best 153 -point design when removing node 35 with power 0.999992515167376
* Best 152 -point design when removing node 11 with power 0.999993140350977
* Best 151 -point design when removing node 121 with power 0.99999360277532
* Best 150 -point design when removing node 111 with power 0.999994158006104
* Best 149 -point design when removing node 111 with power 0.999994638604571
* Best 148 -point design when removing node 28 with power 0.999994992534145
* Best 147 -point design when removing node 45 with power 0.999995314259018
* Best 146 -point design when removing node 60 with power 0.999995610578998
* Best 145 -point design when removing node 52 with power 0.999995882409964
* Best 144 -point design when removing node 39 with power 0.999996172801099
* Best 143 -point design when removing node 51 with power 0.999996597879448
* Best 142 -point design when removing node 61 with power 0.99999680424897
* Best 141 -point design when removing node 41 with power 0.999996997205943
* Best 140 -point design when removing node 30 with power 0.99999722072658
* Best 139 -point design when removing node 31 with power 0.999997489480212
* Best 138 -point design when removing node 52 with power 0.999997780899713
* Best 137 -point design when removing node 50 with power 0.999997973084267
* Best 136 -point design when removing node 58 with power 0.999998118791644
* Best 135 -point design when removing node 71 with power 0.999998276008534
* Best 134 -point design when removing node 70 with power 0.99999846849823
* Best 133 -point design when removing node 119 with power 0.999998561856258
* Best 132 -point design when removing node 126 with power 0.9999987209647
* Best 131 -point design when removing node 108 with power 0.99999882961786
* Best 130 -point design when removing node 73 with power 0.999998899824398
* Best 129 -point design when removing node 82 with power 0.999998992359794
* Best 128 -point design when removing node 71 with power 0.999999089331106
* Best 127 -point design when removing node 90 with power 0.999999142967994
* Best 126 -point design when removing node 99 with power 0.99999920104468
* Best 125 -point design when removing node 80 with power 0.999999281868678
* Best 124 -point design when removing node 78 with power 0.999999364017963
* Best 123 -point design when removing node 76 with power 0.999999412103854
* Best 122 -point design when removing node 93 with power 0.999999457334048
* Best 121 -point design when removing node 101 with power 0.999999504106142
* Best 120 -point design when removing node 99 with power 0.999999540137332
* Best 119 -point design when removing node 84 with power 0.999999575066928
* Best 118 -point design when removing node 85 with power 0.999999621974153
* Best 117 -point design when removing node 43 with power 0.999999644856106
* Best 116 -point design when removing node 52 with power 0.999999682115505
* Best 115 -point design when removing node 44 with power 0.999999730767952
* Best 114 -point design when removing node 52 with power 0.999999770501246
* Best 113 -point design when removing node 41 with power 0.999999789235235
* Best 112 -point design when removing node 59 with power 0.999999803853404
* Best 111 -point design when removing node 60 with power 0.999999821186516
* Best 110 -point design when removing node 67 with power 0.999999829012842
* Best 109 -point design when removing node 75 with power 0.999999842330642
* Best 108 -point design when removing node 65 with power 0.999999848622889
*** Best 107 -point design when removing node 53 with power 0.999999852238676**
* Best 106 -point design when removing node 21 with power 0.99999984911176
* Best 105 -point design when removing node 47 with power 0.99999984576909
* Best 104 -point design when removing node 66 with power 0.99999984238671
* Best 103 -point design when removing node 7 with power 0.999999837031595
* Best 102 -point design when removing node 5 with power 0.999999833361473

* Best 101 -point design when removing node 89 with power 0.99999982785745
* Best 100 -point design when removing node 95 with power 0.999999824111433
* Best 99 -point design when removing node 51 with power 0.999999812018996
* Best 98 -point design when removing node 56 with power 0.999999798872663
* Best 97 -point design when removing node 78 with power 0.999999785081973
* Best 96 -point design when removing node 21 with power 0.9999997677069
* Best 95 -point design when removing node 34 with power 0.99999974282116
* Best 94 -point design when removing node 33 with power 0.999999788288263
* Best 93 -point design when removing node 84 with power 0.999999765772452
* Best 92 -point design when removing node 76 with power 0.999999807441846
* Best 91 -point design when removing node 8 with power 0.999999785718289
* Best 90 -point design when removing node 12 with power 0.999999781738602
* Best 89 -point design when removing node 39 with power 0.999999757076357
* Best 88 -point design when removing node 44 with power 0.999999752533986
* Best 87 -point design when removing node 58 with power 0.999999724655302
* Best 86 -point design when removing node 62 with power 0.999999719453102
* Best 85 -point design when removing node 40 with power 0.999999688037994
* Best 84 -point design when removing node 46 with power 0.999999682382817
* Best 83 -point design when removing node 59 with power 0.999999646959054
* Best 82 -point design when removing node 65 with power 0.99999964068565
* Best 81 -point design when removing node 71 with power 0.999999600745066
* Best 80 -point design when removing node 70 with power 0.999999594026431
* Best 79 -point design when removing node 1 with power 0
* Best 78 -point design when removing node 1 with power 0.999999382838455
* Best 77 -point design when removing node 1 with power 0
* Best 76 -point design when removing node 3 with power 0.999999061967004
* Best 75 -point design when removing node 1 with power 0
* Best 74 -point design when removing node 2 with power 0.999998576620937
* Best 73 -point design when removing node 1 with power 0
* Best 72 -point design when removing node 1 with power 0.99999784419129
* Best 71 -point design when removing node 1 with power 0
* Best 70 -point design when removing node 5 with power 0.999996735052874
* Best 69 -point design when removing node 1 with power 0
* Best 68 -point design when removing node 4 with power 0.999995062096274
* Best 67 -point design when removing node 1 with power 0
* Best 66 -point design when removing node 3 with power 0.999992539689822
* Best 65 -point design when removing node 1 with power 0
* Best 64 -point design when removing node 1 with power 0.999988749729435
* Best 63 -point design when removing node 1 with power 0
* Best 62 -point design when removing node 1 with power 0.999983060491738
* Best 61 -point design when removing node 1 with power 0
* Best 60 -point design when removing node 5 with power 0.999974500050425
* Best 59 -point design when removing node 1 with power 0
* Best 58 -point design when removing node 1 with power 0.999961699864542
* Best 57 -point design when removing node 1 with power 0
* Best 56 -point design when removing node 1 with power 0.99994259805925
* Best 55 -point design when removing node 1 with power 0
* Best 54 -point design when removing node 1 with power 0.999913988982287
* Best 53 -point design when removing node 1 with power 0
* Best 52 -point design when removing node 6 with power 0.99987138912728
* Best 51 -point design when removing node 1 with power 0
* Best 50 -point design when removing node 5 with power 0.99980805603154
* Best 49 -point design when removing node 1 with power 0
* Best 48 -point design when removing node 1 with power 0.999714081107554
* Best 47 -point design when removing node 1 with power 0
* Best 46 -point design when removing node 1 with power 0.99957475820725
* Best 45 -point design when removing node 1 with power 0

* Best 44 -point design when removing node 1 with power 0.999369034953871
* Best 43 -point design when removing node 1 with power 0
* Best 42 -point design when removing node 1 with power 0.999065634920424
* Best 41 -point design when removing node 1 with power 0
* Best 40 -point design when removing node 6 with power 0.99861982169275
* Best 39 -point design when removing node 1 with power 0
* Best 38 -point design when removing node 1 with power 0.997965541980496
* Best 37 -point design when removing node 1 with power 0
* Best 36 -point design when removing node 5 with power 0.997011031089992
* Best 35 -point design when removing node 1 with power 0
* Best 34 -point design when removing node 1 with power 0.995618359634806
* Best 33 -point design when removing node 1 with power 0
* Best 32 -point design when removing node 3 with power 0.993595632427806
* Best 31 -point design when removing node 1 with power 0
* Best 30 -point design when removing node 3 with power 0.99066851555673
* Best 29 -point design when removing node 1 with power 0
* Best 28 -point design when removing node 4 with power 0.986451195393454
* Best 27 -point design when removing node 1 with power 0
* Best 26 -point design when removing node 3 with power 0.980406885665569
* Best 25 -point design when removing node 1 with power 0
* Best 24 -point design when removing node 1 with power 0.97177113645725
* Best 23 -point design when removing node 1 with power 0
* Best 22 -point design when removing node 1 with power 0.959525325322036
* Best 21 -point design when removing node 1 with power 0
* Best 20 -point design when removing node 1 with power 0.942274191347537
* Best 19 -point design when removing node 1 with power 0
* Best 18 -point design when removing node 1 with power 0.918117925906051
* Best 17 -point design when removing node 1 with power 0
* Best 16 -point design when removing node 1 with power 0.884603382375019
* Best 15 -point design when removing node 1 with power 0
* Best 14 -point design when removing node 5 with power 0.838540181898833
* Best 13 -point design when removing node 1 with power 0
* Best 12 -point design when removing node 1 with power 0.776168077764942
* Best 11 -point design when removing node 1 with power 0
* Best 10 -point design when removing node 1 with power 0.692954793822438
* Best 9 -point design when removing node 1 with power 0
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* Best 7 -point design when removing node 1 with power 0
* Best 6 -point design when removing node 1 with power 0.448161307881256
* Best 5 -point design when removing node 1 with power 0
* Best 4 -point design when removing node 1 with power 0.287964783357575
* Best 3 -point design when removing node 1 with power 0
* Best 2 -point design when removing node 1 with power 0

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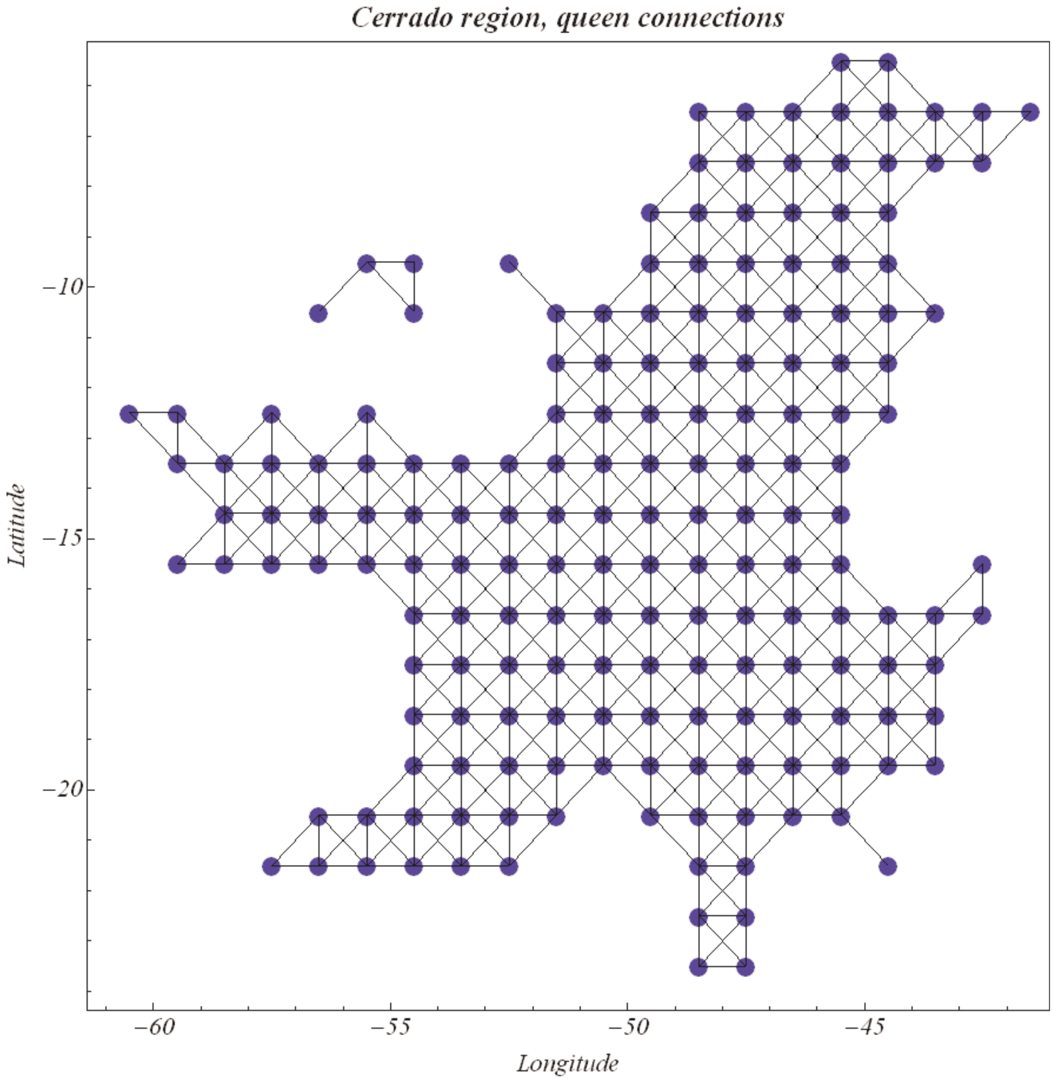
CERRADO (BRAZIL): SIMPLE SEARCH ALGORITHM, RHO=0.5, QUEEN'S RULE



Complete design (181 nodes) power: 0.9963748

* Best 180 -point design when removing node 79 with power 0.99668133642746
* Best 179 -point design when removing node 65 with power 0.996967160877793
* Best 178 -point design when removing node 66 with power 0.997181406996482
* Best 177 -point design when removing node 78 with power 0.997393731182428
* Best 176 -point design when removing node 77 with power 0.997779698316177
* Best 175 -point design when removing node 90 with power 0.997905182219492
* Best 174 -point design when removing node 114 with power 0.998013368627344
* Best 173 -point design when removing node 126 with power 0.99825093070045
* Best 172 -point design when removing node 77 with power 0.998334921400795
* Best 171 -point design when removing node 77 with power 0.998446112870066
* Best 170 -point design when removing node 67 with power 0.998652799011278
* Best 169 -point design when removing node 8 with power 0.998707651474563
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* Best 163 -point design when removing node 145 with power 0.999114590081418
* Best 162 -point design when removing node 132 with power 0.999147196535889
* Best 161 -point design when removing node 154 with power 0.999199127809552

* Best 160 -point design when removing node 131 with power 0.99927111581342
* Best 159 -point design when removing node 6 with power 0.999297822630857
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* Best 156 -point design when removing node 5 with power 0.99943409798829
* Best 155 -point design when removing node 16 with power 0.99945532303866
* Best 154 -point design when removing node 45 with power 0.99947589398648
* Best 153 -point design when removing node 135 with power 0.999495305750655
* Best 152 -point design when removing node 7 with power 0.999511444507302
* Best 151 -point design when removing node 6 with power 0.999546784222805
* Best 150 -point design when removing node 6 with power 0.999598988931542
* Best 149 -point design when removing node 13 with power 0.999617399645616
* Best 148 -point design when removing node 12 with power 0.999663527255333
* Best 147 -point design when removing node 16 with power 0.999676127910833
* Best 146 -point design when removing node 15 with power 0.999689869804068
* Best 145 -point design when removing node 15 with power 0.999708199637045
* Best 144 -point design when removing node 15 with power 0.999725215524103
* Best 143 -point design when removing node 29 with power 0.999736118779496
* Best 142 -point design when removing node 59 with power 0.999744794678055
* Best 141 -point design when removing node 50 with power 0.999755570859416
* Best 140 -point design when removing node 70 with power 0.99976952305072
* Best 139 -point design when removing node 69 with power 0.999789599817642
* Best 138 -point design when removing node 89 with power 0.999796994704286
* Best 137 -point design when removing node 99 with power 0.999812086701097
* Best 136 -point design when removing node 77 with power 0.99982838387608
* Best 135 -point design when removing node 120 with power 0.999834673874175
* Best 134 -point design when removing node 109 with power 0.999842388497976
* Best 133 -point design when removing node 108 with power 0.999856588823737
* Best 132 -point design when removing node 89 with power 0.999862225471281
* Best 131 -point design when removing node 98 with power 0.999868526423452
* Best 130 -point design when removing node 78 with power 0.999876393918442
* Best 129 -point design when removing node 77 with power 0.999883433892469
* Best 128 -point design when removing node 126 with power 0.99988730541458
* Best 127 -point design when removing node 126 with power 0.999903352253584
* Best 126 -point design when removing node 102 with power 0.99990650200886
* Best 125 -point design when removing node 109 with power 0.999913732112313
* Best 124 -point design when removing node 93 with power 0.999924366262702
* Best 123 -point design when removing node 106 with power 0.999927528114216
* Best 122 -point design when removing node 113 with power 0.999931130140678
* Best 121 -point design when removing node 106 with power 0.999937276375075
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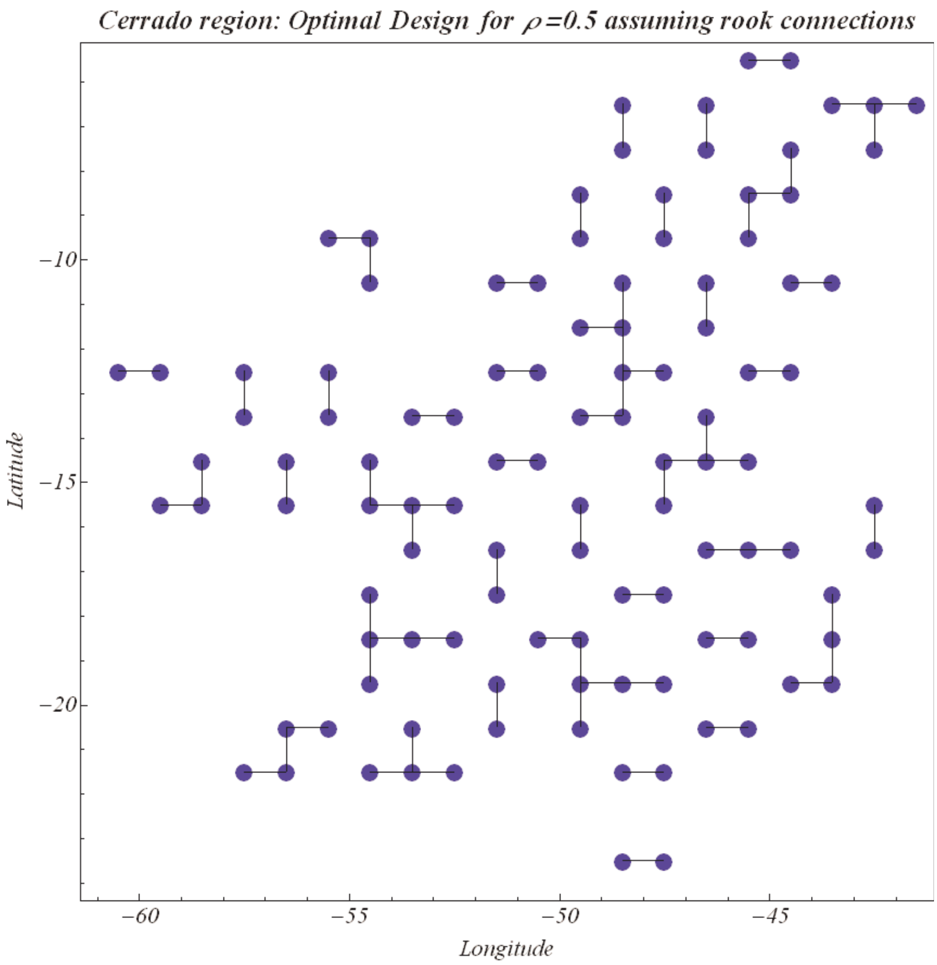
* Best 117 -point design when removing node 96 with power 0.999951867879228
* Best 116 -point design when removing node 96 with power 0.999953765284845
* Best 115 -point design when removing node 95 with power 0.999956491830338
* Best 114 -point design when removing node 101 with power 0.999960520818974
* Best 113 -point design when removing node 96 with power 0.9999625197941
* Best 112 -point design when removing node 91 with power 0.999965457545945
* Best 111 -point design when removing node 81 with power 0.999967102073085
* Best 110 -point design when removing node 88 with power 0.9999688641547
* Best 109 -point design when removing node 88 with power 0.999971671984719
* Best 108 -point design when removing node 74 with power 0.999972790852364
* Best 107 -point design when removing node 77 with power 0.999973831559369
* Best 106 -point design when removing node 76 with power 0.999975853230075
* Best 105 -point design when removing node 76 with power 0.999979064831411
* Best 104 -point design when removing node 59 with power 0.999980090650604

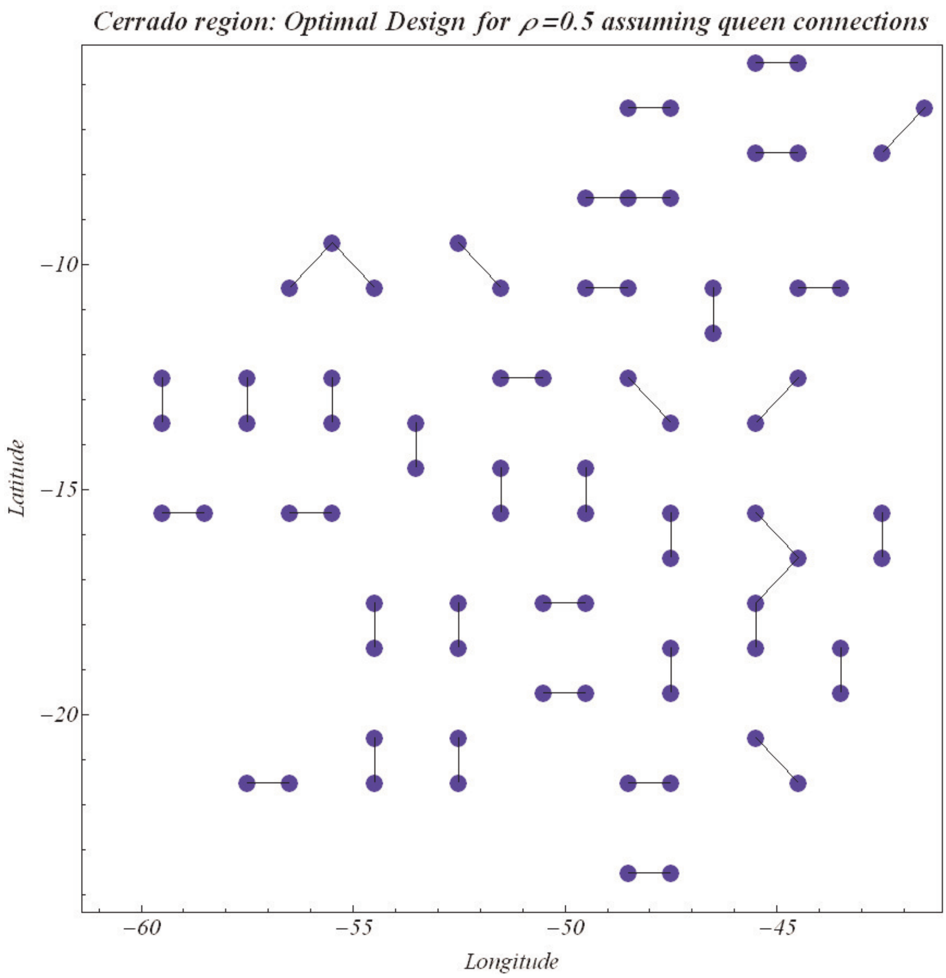
* Best 103 -point design when removing node 71 with power 0.99998099986254
* Best 102 -point design when removing node 62 with power 0.999981985164905
* Best 101 -point design when removing node 60 with power 0.99998331785285
* Best 100 -point design when removing node 60 with power 0.999985205410757
* Best 99 -point design when removing node 66 with power 0.999986105886466
* Best 98 -point design when removing node 51 with power 0.99998745014433
* Best 97 -point design when removing node 50 with power 0.999988743907081
* Best 96 -point design when removing node 28 with power 0.999989452618746
* Best 95 -point design when removing node 20 with power 0.99999044870132
* Best 94 -point design when removing node 48 with power 0.999990935916968
* Best 93 -point design when removing node 48 with power 0.999992134242083
* Best 92 -point design when removing node 27 with power 0.999992681345874
* Best 91 -point design when removing node 27 with power 0.999993437591369
* Best 90 -point design when removing node 27 with power 0.99999399929813
* Best 89 -point design when removing node 22 with power 0.99999457487619
* Best 88 -point design when removing node 34 with power 0.999994949808378
* Best 87 -point design when removing node 27 with power 0.9999952194138
* Best 86 -point design when removing node 23 with power 0.999995504101695
* Best 85 -point design when removing node 33 with power 0.999995777898353
* Best 84 -point design when removing node 34 with power 0.999996103858167
* Best 83 -point design when removing node 33 with power 0.999996573469863
* Best 82 -point design when removing node 15 with power 0.999997021025213
* Best 81 -point design when removing node 38 with power 0.999997409087348
* Best 80 -point design when removing node 25 with power 0.999997680232989
* Best 79 -point design when removing node 41 with power 0.99999788103297
* Best 78 -point design when removing node 51 with power 0.99999802889067
* Best 77 -point design when removing node 15 with power 0.9999981461285
*** Best 76 -point design when removing node 13 with power 0.999998153329114**
* Best 75 -point design when removing node 9 with power 0.99999812117047
* Best 74 -point design when removing node 13 with power 0.999998088678159
* Best 73 -point design when removing node 44 with power 0.999997879179584
* Best 72 -point design when removing node 46 with power 0.99999784419129
* Best 71 -point design when removing node 1 with power 0
* Best 70 -point design when removing node 1 with power 0.999996735052874
* Best 69 -point design when removing node 1 with power 0
* Best 68 -point design when removing node 1 with power 0.999995062096274
* Best 67 -point design when removing node 1 with power 0
* Best 66 -point design when removing node 3 with power 0.999992539689822
* Best 65 -point design when removing node 1 with power 0
* Best 64 -point design when removing node 1 with power 0.999988749729435
* Best 63 -point design when removing node 1 with power 0
* Best 62 -point design when removing node 1 with power 0.999983060491738
* Best 61 -point design when removing node 1 with power 0
* Best 60 -point design when removing node 2 with power 0.999974500050425
* Best 59 -point design when removing node 1 with power 0
* Best 58 -point design when removing node 1 with power 0.999961699864542
* Best 57 -point design when removing node 1 with power 0
* Best 56 -point design when removing node 1 with power 0.99994259805925
* Best 55 -point design when removing node 1 with power 0
* Best 54 -point design when removing node 3 with power 0.999913988982287
* Best 53 -point design when removing node 1 with power 0
* Best 52 -point design when removing node 1 with power 0.99987138912728
* Best 51 -point design when removing node 1 with power 0
* Best 50 -point design when removing node 7 with power 0.99980805603154
* Best 49 -point design when removing node 1 with power 0
* Best 48 -point design when removing node 6 with power 0.999714081107554
* Best 47 -point design when removing node 1 with power 0

* Best 46 -point design when removing node 5 with power 0.99957475820725
* Best 45 -point design when removing node 1 with power 0
* Best 44 -point design when removing node 1 with power 0.999369034953871
* Best 43 -point design when removing node 1 with power 0
* Best 42 -point design when removing node 3 with power 0.999065634920424
* Best 41 -point design when removing node 1 with power 0
* Best 40 -point design when removing node 2 with power 0.99861982169275
* Best 39 -point design when removing node 1 with power 0
* Best 38 -point design when removing node 1 with power 0.997965541980496
* Best 37 -point design when removing node 1 with power 0
* Best 36 -point design when removing node 6 with power 0.997011031089992
* Best 35 -point design when removing node 1 with power 0
* Best 34 -point design when removing node 5 with power 0.995618359634806
* Best 33 -point design when removing node 1 with power 0
* Best 32 -point design when removing node 1 with power 0.993595632427806
* Best 31 -point design when removing node 1 with power 0
* Best 30 -point design when removing node 1 with power 0.99066851555673
* Best 29 -point design when removing node 1 with power 0
* Best 28 -point design when removing node 2 with power 0.986451195393454
* Best 27 -point design when removing node 1 with power 0
* Best 26 -point design when removing node 1 with power 0.980406885665569
* Best 25 -point design when removing node 1 with power 0
* Best 24 -point design when removing node 5 with power 0.97177113645725
* Best 23 -point design when removing node 1 with power 0
* Best 22 -point design when removing node 4 with power 0.959525325322036
* Best 21 -point design when removing node 1 with power 0
* Best 20 -point design when removing node 1 with power 0.942274191347538
* Best 19 -point design when removing node 1 with power 0
* Best 18 -point design when removing node 2 with power 0.91811792590605
* Best 17 -point design when removing node 1 with power 0
* Best 16 -point design when removing node 4 with power 0.88460338237502
* Best 15 -point design when removing node 1 with power 0
* Best 14 -point design when removing node 3 with power 0.838540181898832
* Best 13 -point design when removing node 1 with power 0
* Best 12 -point design when removing node 1 with power 0.776168077764942
* Best 11 -point design when removing node 1 with power 0
* Best 10 -point design when removing node 5 with power 0.692954793822439
* Best 9 -point design when removing node 1 with power 0
* Best 8 -point design when removing node 4 with power 0.58448953056749
* Best 7 -point design when removing node 1 with power 0
* Best 6 -point design when removing node 5 with power 0.448161307881256
* Best 5 -point design when removing node 1 with power 0
* Best 4 -point design when removing node 1 with power 0.287964783357575
* Best 3 -point design when removing node 1 with power 0
* Best 2 -point design when removing node 1 with power 0

****
****

OPTIMAL DESIGNS





COMMENTS:

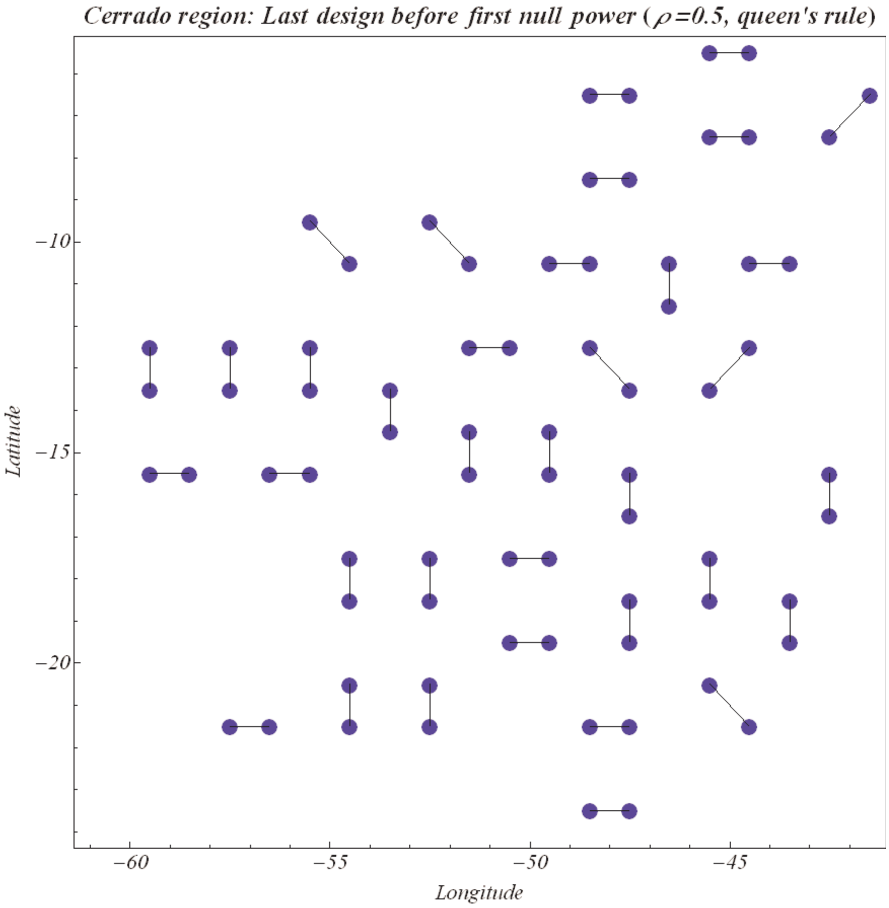
* The number of the node removed every time refers to the remaining nodes in the design at that point, thus it is not the actual number of the region. The actual numbers are respectively:

{177,33,26,120,7,6,132,162,155,144,166,154,179,178,165,19,12,4,21,14,37,28,45,39,30,41,32,51,16,141,130,131,44,64,80,72,58,73,85,61,48,50,78,76,88,102,101,160,171,147,106,116,104,127,140,115,113,111,137,150,148,124,126,67,82,69,84,65,95,97,109,123,107,89,34,79,117,10,8,161,172,92,100,146,38,60,59,164,151,15,23,77,91,122,134,83,98,133,145,159,158,1,2,3,11,5,13,9,17,18,27,20,29,22,31,24,25,35,36,40,49,42,43,46,47,52,53,54,66,55,68,56,57,62,63,70,71,74,75,81,96,86,87,90,105,93,94,99,110,103,114,108,121,112,125,118,119,128,129,135,136,138,139,142,143,149,163,152,153,156,157,167,168,169,170,173,174,175,176}
for the rook case (including the 3 isolated ones at the beginning), and

{79,65,67,81,80,95,120,133,82,83,69,8,16,109,98,159,171,161,147,173,146,6,7,5,9,22,52,158,12,11,13,23,21,28,27,29,30,45,85,71,100,99,123,135,110,163,149,148,125,137,112,111,178,179,144,156,132,154,167,155,165,166,164,139,140,138,152,142,130,117,128,129,106,114,113,115,87,104,91,89,90,102,73,72,44,36,74,75,46,47,48,39,58,50,41,60,62,61,31,77,51,92,118,32,25,18,33,107,119,1,2,3,4,10,17,14,15,19,20,24,34,26,35,37,38,40,49,42,43,53,64,54,66,55,68,56,57,59,76,63,78,70,84,86,101,88,103,93,94,96,97,105,116,108,121,122,134,124,136,126,127,131,143,141,153,145,157,150,151,160,172,162,174,168,177,169,170,175,176}
for queen connections.

*** The (quasi)-optimal design found using this procedure is a 107 -point design for rook connections, with power 0.999999852238676. For queen connections the (quasi)-optimal design obtained using Simple Search is a 76-point design obtained after removing the first 105 nodes of the above list, with power 0.999998153329114 There is a big difference in number of nodes for the two situations.**

* The reason of the 0 powers after the optimal design is the following: after removing many nodes we get to a point where the remaining regions in the design are connected just in pairs. Thus in each step one of the components of the pair is removed, giving a non-connected region and thus a 0 power; and then, in the following step, the isolate region is removed and again a non-zero power is obtained, and so on. To illustrate this fact, see the 72-point design for the queen's rule, the previous to the first 0 power and very close to the optimal design:



* The results after beginning to obtain zero powers are not very trustable, since for those cases every node removed gives 0 power and thus the first node is always the one chosen to be removed. But of course, all these things happen after the optimal design is found, thus they have a relative importance.

* The powers obtained are quite high, specially when thinking that we have used rho=0.5 , while in the paper the best model corresponds to rho= 0.921 (rook connections) or rho= 0.893 (queen connections).

IDEAS FOR FUTURE WORK:

* Repeat the computations using the normal approximation in order to compare the designs obtained using both procedures. If the idea of the paper is to show that both exact and normal-approximation distributions are almost equivalent for obtaining the optimal designs, this will be necessary.

* Compare the optimal designs for the two types of connections (rook and queen) and compute efficiencies (????)

EXCHANGE ALGORITHM

Taking as base.design the optimal design found using the Simple Search procedure, let us try to improve the power using the exchange algorithm:

ROOK CONNECTIONS

#best.design.ssearch.rook <- c(1 ,2 ,3 ,5 ,8 ,9 ,10 ,11 ,13 ,
#15 ,17 ,18 ,20 ,22 ,23 ,24 ,25 ,27 ,29 ,31 ,34 ,35 ,36 ,38 ,40 ,
#42 ,43 ,46 ,47 ,49 ,52 ,53 ,54 ,55 ,56 ,57 ,59 ,60 ,62 ,63 ,
#66 ,68 ,70 ,71 ,74 ,75 ,77 ,79 ,81 ,83 ,86 ,87 ,90 ,91 ,
#92 ,93 ,94 ,96 ,98 ,99 ,100 ,103 ,105 ,108 ,110 ,112 ,114 ,117 ,118 ,
#119 ,121 ,122 ,125 ,128 ,129 ,133 ,134 ,135 ,136 ,138 ,
#139 ,142 ,143 ,145 ,146 ,149 ,151 ,152 ,153 ,156 ,157 ,158 ,159 ,161 ,163 ,
#164 ,167 ,168 ,169 ,170 ,172 ,173 ,174 ,175 ,176 ,180 ,181)

better power (15,6): 0.999999852333568 --- (exchanging the 15^{th} point in the base.design by the 6^{th} point of the set of candidate.points = points not in the base design)

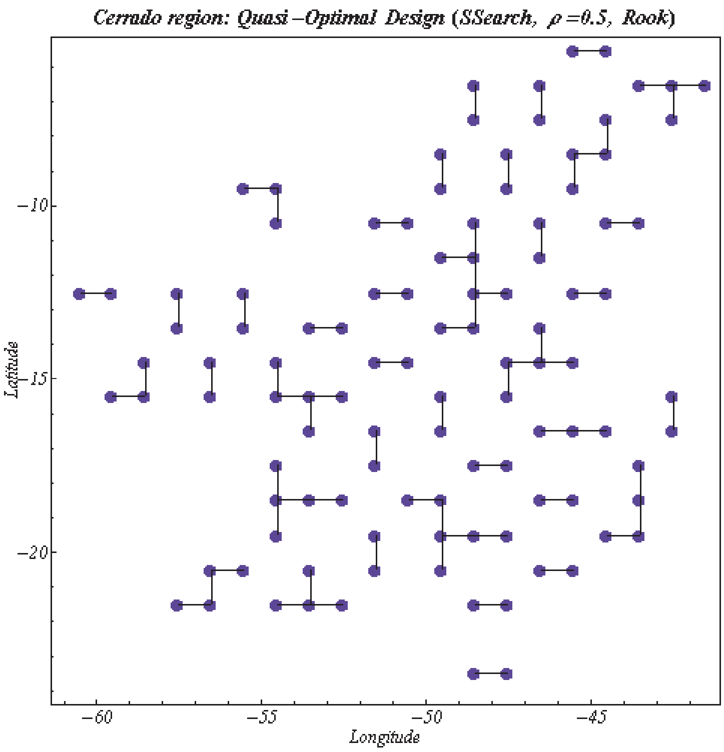
* Base design: 1, 2, 3, 5, 8, 9, 10, 11, 13, 15, 17, 18, 20, 22, 23, 24, 25, 27, 29, 31, 34, 35, 36, 38, 40, 42, 43, 46, 47, 49, 52, 53, 54, 55, 56, 57, 59, 60, 62, 63, 66, 68, 70, 71, 74, 75, 77, 79, 81, 83, 86, 87, 90, 91, 92, 93, 94, 96, 98, 99, 100, 103, 105, 108, 110, 112, 114, 117, 118, 119, 121, 122, 125, 128, 129, 133, 134, 135, 136, 138, 139, 142, 143, 145, 146, 149, 151, 152, 153, 156, 157, 158, 159, 161, 163, 164, 167, 168, 169, 170, 172, 173, 174, 175, 176, 180, 181
with power: 0.999999852238676 .
Candidates: 4, 6, 7, 12, 14, 16, 19, 21, 26, 28, 30, 32, 33, 37, 39, 41, 44, 45, 48, 50, 51, 58, 61, 64, 65, 67, 69, 72, 73, 76, 78, 80, 82, 84, 85, 88, 89, 95, 97, 101, 102, 104, 106, 107, 109, 111, 113, 115, 116, 120, 123, 124, 126, 127, 130, 131, 132, 137, 140, 141, 144, 147, 148, 150, 154, 155, 160, 162, 165, 166, 171, 177, 178, 179
- better power (15,6): 0.999999852333568 in design 1, 2, 3, 5, 8, 9, 10, 11, 13, 15, 16, 17, 18, 20, 22, 24, 25, 27, 29, 31, 34, 35, 36, 38, 40, 42, 43, 46, 47, 49, 52, 53, 54, 55, 56, 57, 59, 60, 62, 63, 66, 68, 70, 71, 74, 75, 77, 79, 81, 83, 86, 87, 90, 91, 92, 93, 94, 96, 98, 99, 100, 103, 105, 108, 110, 112, 114, 117, 118, 119, 121, 122, 125, 128, 129, 133, 134, 135, 136, 138, 139, 142, 143, 145, 146, 149, 151, 152, 153, 156, 157, 158, 159, 161, 163, 164, 167, 168, 169, 170, 172, 173, 174, 175, 176, 180, 181
- better power (70,57): 0.999999863396202 in design 1, 2, 3, 5, 8, 9, 10, 11, 13, 15, 17, 18, 20, 22, 23, 24, 25, 27, 29, 31, 34, 35, 36, 38, 40, 42, 43, 46, 47, 49, 52, 53, 54, 55, 56, 57, 59, 60, 62, 63, 66, 68, 70, 71, 74, 75, 77, 79, 81, 83, 86, 87, 90, 91, 92, 93, 94, 96, 98, 99, 100, 103, 105, 108, 110, 112, 114, 117, 118, 121, 122, 125, 128, 129, 132, 133, 134, 135, 136, 138, 139, 142, 143, 145, 146, 149, 151, 152, 153, 156, 157, 158, 159, 161, 163, 164, 167, 168, 169, 170, 172, 173, 174, 175, 176, 180, 181

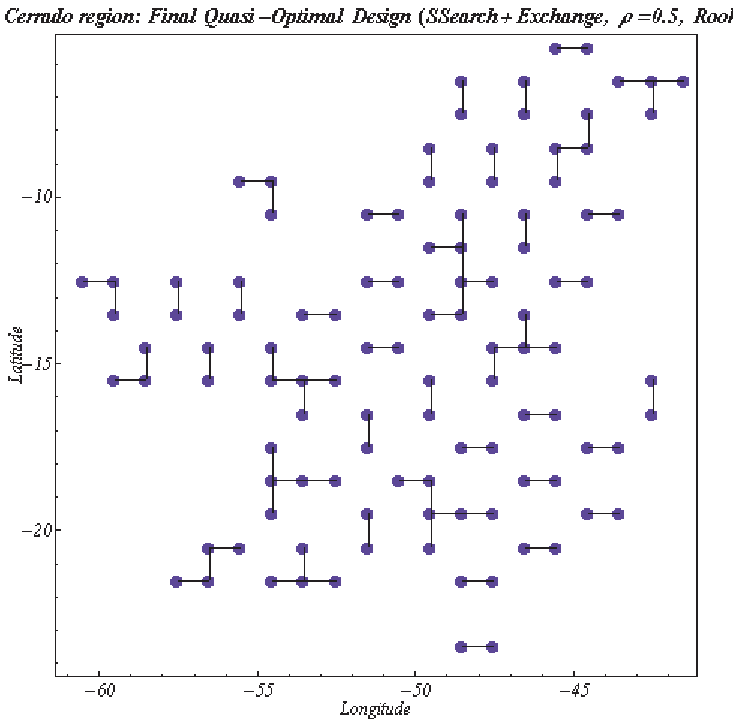
* Base design: 1, 2, 3, 5, 8, 9, 10, 11, 13, 15, 17, 18, 20, 22, 23, 24, 25, 27, 29, 31, 34, 35, 36, 38, 40, 42, 43, 46, 47, 49, 52, 53, 54, 55, 56, 57, 59, 60, 62, 63, 66, 68, 70, 71, 74, 75, 77, 79, 81, 83, 86, 87, 90, 91, 92, 93, 94, 96, 98, 99, 100, 103, 105, 108, 110, 112, 114, 117, 118, 121, 122, 125, 128, 129, 132, 133, 134, 135, 136, 138, 139, 142, 143, 145, 146, 149, 151, 152, 153, 156, 157, 158, 159, 161, 163, 164, 167, 168, 169, 170, 172, 173, 174, 175, 176, 180, 181
with power: 0.999999863396202 .
Candidates: 4, 6, 7, 12, 14, 16, 19, 21, 26, 28, 30, 32, 33, 37, 39, 41, 44, 45, 48, 50, 51, 58, 61, 64, 65, 67, 69, 72, 73, 76, 78, 80, 82, 84, 85, 88, 89, 95, 97, 101, 102, 104, 106, 107, 109, 111, 113, 115, 116, 119, 120, 123, 124, 126, 127, 130, 131, 137, 140, 141, 144, 147, 148, 150, 154, 155, 160, 162, 165, 166, 171, 177, 178, 179
- better power (15,6): 0.999999863500719 in design 1, 2, 3, 5, 8, 9, 10, 11, 13, 15, 16, 17, 18, 20, 22, 24, 25, 27, 29, 31, 34, 35, 36, 38, 40, 42, 43, 46, 47, 49, 52, 53, 54, 55, 56, 57, 59, 60, 62, 63, 66, 68, 70, 71, 74, 75, 77, 79, 81, 83, 86, 87, 90, 91, 92, 93, 94, 96, 98, 99, 100, 103, 105, 108, 110, 112, 114, 117, 118, 121, 122, 125, 128, 129, 132, 133, 134, 135, 136, 138, 139, 142, 143, 145, 146, 149, 151, 152, 153, 156, 157, 158, 159, 161, 163, 164, 167, 168, 169, 170, 172, 173, 174, 175, 176, 180, 181
- better power (84,6): 0.999999876054987 in design 1, 2, 3, 5, 8, 9, 10, 11, 13, 15, 16, 17, 18, 20, 22, 23, 24, 25, 27, 29, 31, 34, 35, 36, 38, 40, 42, 43, 46, 47, 49, 52, 53, 54, 55, 56, 57, 59, 60, 62, 63, 66, 68, 70, 71, 74, 75, 77, 79, 81, 83, 86, 87, 90, 91, 92, 93, 94, 96, 98, 99, 100, 103, 105, 108, 110, 112, 114, 117, 118, 121, 122, 125, 128, 129, 132, 133, 134, 135, 136, 138, 139, 142, 143, 146, 149, 151, 152, 153, 156, 157, 158, 159, 161, 163, 164, 167, 168, 169, 170, 172, 173, 174, 175, 176, 180, 181

- better power (84,24): 0.999999889904636 in design 1, 2, 3, 5, 8, 9, 10, 11, 13, 15, 17, 18, 20, 22, 23, 24, 25, 27, 29, 31, 34, 35, 36, 38, 40, 42, 43, 46, 47, 49, 52, 53, 54, 55, 56, 57, 59, 60, 62, 63, 64, 66, 68, 70, 71, 74, 75, 77, 79, 81, 83, 86, 87, 90, 91, 92, 93, 94, 96, 98, 99, 100, 103, 105, 108, 110, 112, 114, 117, 118, 121, 122, 125, 128, 129, 132, 133, 134, 135, 136, 138, 139, 142, 143, 146, 149, 151, 152, 153, 156, 157, 158, 159, 161, 163, 164, 167, 168, 169, 170, 172, 173, 174, 175, 176, 180, 181

* Base design: 1, 2, 3, 5, 8, 9, 10, 11, 13, 15, 17, 18, 20, 22, 23, 24, 25, 27, 29, 31, 34, 35, 36, 38, 40, 42, 43, 46, 47, 49, 52, 53, 54, 55, 56, 57, 59, 60, 62, 63, 64, 66, 68, 70, 71, 74, 75, 77, 79, 81, 83, 86, 87, 90, 91, 92, 93, 94, 96, 98, 99, 100, 103, 105, 108, 110, 112, 114, 117, 118, 121, 122, 125, 128, 129, 132, 133, 134, 135, 136, 138, 139, 142, 143, 146, 149, 151, 152, 153, 156, 157, 158, 159, 161, 163, 164, 167, 168, 169, 170, 172, 173, 174, 175, 176, 180, 181
with power: 0.999999889904636 .
Candidates: 4, 6, 7, 12, 14, 16, 19, 21, 26, 28, 30, 32, 33, 37, 39, 41, 44, 45, 48, 50, 51, 58, 61, 65, 67, 69, 72, 73, 76, 78, 80, 82, 84, 85, 88, 89, 95, 97, 101, 102, 104, 106, 107, 109, 111, 113, 115, 116, 119, 120, 123, 124, 126, 127, 130, 131, 137, 140, 141, 144, 145, 147, 148, 150, 154, 155, 160, 162, 165, 166, 171, 177, 178, 179
There is no improvement by exchanging points

***** Best design after 3 iterations: 1, 2, 3, 5, 8, 9, 10, 11, 13, 15, 17, 18, 20, 22, 23, 24, 25, 27, 29, 31, 34, 35, 36, 38, 40, 42, 43, 46, 47, 49, 52, 53, 54, 55, 56, 57, 59, 60, 62, 63, 64, 66, 68, 70, 71, 74, 75, 77, 79, 81, 83, 86, 87, 90, 91, 92, 93, 94, 96, 98, 99, 100, 103, 105, 108, 110, 112, 114, 117, 118, 121, 122, 125, 128, 129, 132, 133, 134, 135, 136, 138, 139, 142, 143, 146, 149, 151, 152, 153, 156, 157, 158, 159, 161, 163, 164, 167, 168, 169, 170, 172, 173, 174, 175, 176, 180, 181
with power: 0.999999889904636

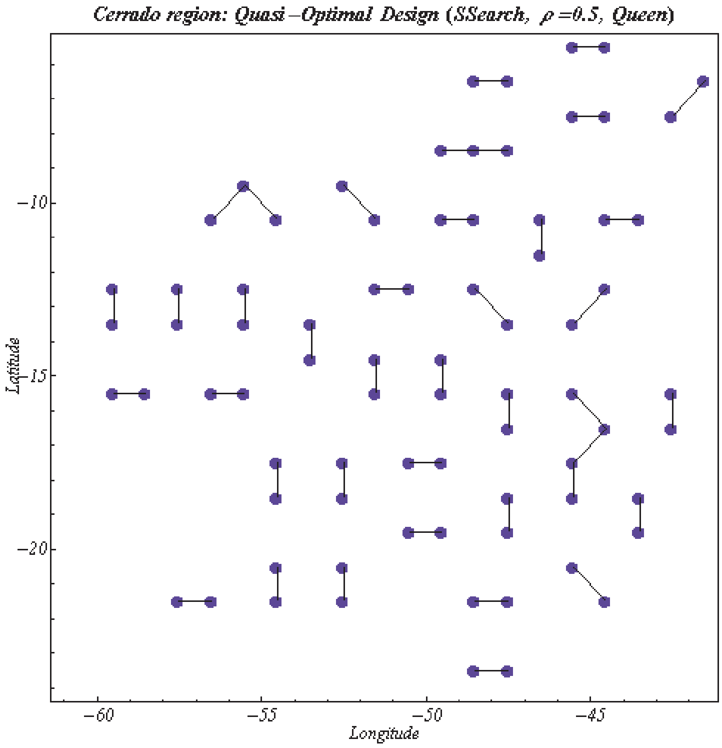


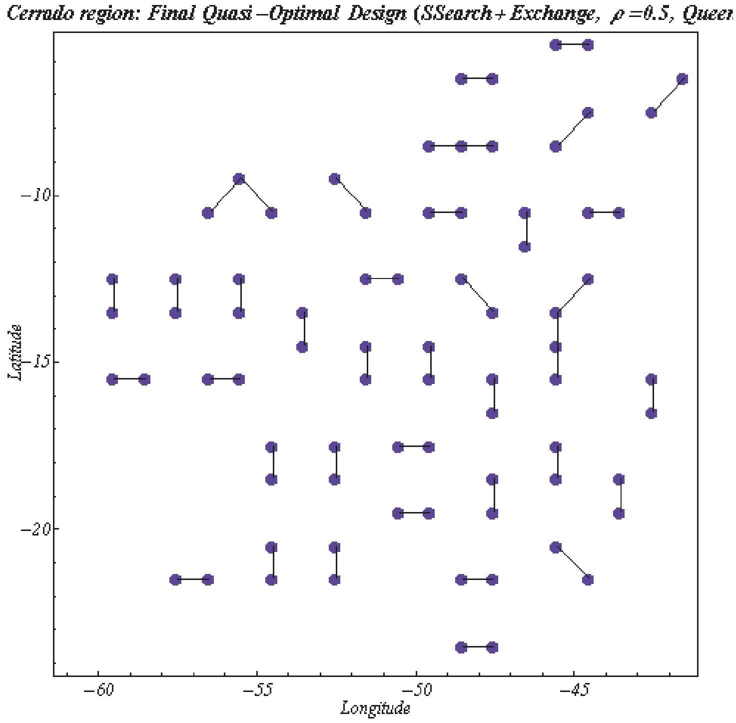


QUEEN’S RULE

* Base design: 1, 2, 3, 4, 10, 14, 15, 17, 18, 19, 20, 24, 26, 33, 34, 35, 37, 38, 40, 42, 43, 49, 53, 54, 55, 56, 57, 59, 63, 64, 66, 68, 70, 76, 78, 84, 86, 88, 93, 94, 96, 97, 101, 103, 105, 107, 108, 116, 119, 121, 122, 124, 126, 127, 131, 134, 136, 141, 143, 145, 150, 151, 153, 157, 160, 162, 168, 169, 170, 172, 174, 175, 176, 177, 180, 181
with power: 0.999998153329114 .
Candidates: 5, 6, 7, 8, 9, 11, 12, 13, 16, 21, 22, 23, 25, 27, 28, 29, 30, 31, 32, 36, 39, 41, 44, 45, 46, 47, 48, 50, 51, 52, 58, 60, 61, 62, 65, 67, 69, 71, 72, 73, 74, 75, 77, 79, 80, 81, 82, 83, 85, 87, 89, 90, 91, 92, 95, 98, 99, 100, 102, 104, 106, 109, 110, 111, 112, 113, 114, 115, 117, 118, 120, 123, 125, 128, 129, 130, 132, 133, 135, 137, 138, 139, 140, 142, 144, 146, 147, 148, 149, 152, 154, 155, 156, 158, 159, 161, 163, 164, 165, 166, 167, 171, 173, 178, 179
    - better power (6,11): 0.999998153329114 in design 1, 2, 3, 4, 10, 15, 17, 18, 19, 20, 22, 24, 26, 33, 34, 35, 37, 38, 40, 42, 43, 49, 53, 54, 55, 56, 57, 59, 63, 64, 66, 68, 70, 76, 78, 84, 86, 88, 93, 94, 96, 97, 101, 103, 105, 107, 108, 116, 119, 121, 122, 124, 126, 127, 131, 134, 136, 141, 143, 145, 150, 151, 153, 157, 160, 162, 168, 169, 170, 172, 174, 175, 176, 177, 180, 181

* Base design: 1, 2, 3, 4, 10, 15, 17, 18, 19, 20, 22, 24, 26, 33, 34, 35, 37, 38, 40, 42, 43, 49, 53, 54, 55, 56, 57, 59, 63, 64, 66, 68, 70, 76, 78, 84, 86, 88, 93, 94, 96, 97, 101, 103, 105, 107, 108, 116, 119, 121, 122, 124, 126, 127, 131, 134, 136, 141, 143, 145, 150, 151, 153, 157, 160, 162, 168, 169, 170, 172, 174, 175, 176, 177, 180, 181
with power: 0.999998153329114 .
Candidates: 5, 6, 7, 8, 9, 11, 12, 13, 14, 16, 21, 23, 25, 27, 28, 29, 30, 31, 32, 36, 39, 41, 44, 45, 46, 47, 48, 50, 51, 52, 58, 60, 61, 62, 65, 67, 69, 71, 72, 73, 74, 75, 77, 79, 80, 81, 82, 83, 85, 87, 89, 90, 91, 92, 95, 98, 99, 100, 102, 104, 106, 109, 110, 111, 112, 113, 114, 115, 117, 118, 120, 123, 125, 128, 129, 130, 132, 133, 135, 137, 138, 139, 140, 142, 144, 146, 147, 148, 149, 152, 154, 155, 156, 158, 159, 161, 163, 164, 165, 166, 167, 171, 173, 178, 179
    - better power (49,54): 0.999998153329114 in design 1, 2, 3, 4, 10, 15, 17, 18, 19, 20, 22, 24, 26, 33, 34, 35, 37, 38, 40, 42, 43, 49, 53, 54, 55, 56, 57, 59, 63, 64, 66, 68, 70, 76, 78, 84, 86, 88, 92, 93, 94, 96, 97, 101, 103, 105, 107, 108, 116, 121, 122, 124, 126, 127, 131, 134, 136, 141, 143, 145, 150, 151, 153, 157, 160, 162, 168, 169, 170, 172, 174, 175, 176, 177, 180, 181
* Base design: 1, 2, 3, 4, 10, 15, 17, 18, 19, 20, 22, 24, 26, 33, 34, 35, 37, 38, 40, 42, 43, 49, 53, 54, 55, 56, 57, 59, 63, 64, 66, 68, 70, 76, 78, 84, 86, 88, 92, 93, 94, 96, 97, 101, 103, 105, 107, 108, 116, 121, 122, 124, 126, 127, 131, 134, 136, 141, 143, 145, 150, 151, 153, 157, 160, 162, 168, 169, 170, 172, 174, 175, 176, 177, 180, 181
with power: 0.999998153329114 .
Candidates: 5, 6, 7, 8, 9, 11, 12, 13, 14, 16, 21, 23, 25, 27, 28, 29, 30, 31, 32, 36, 39, 41, 44, 45, 46, 47, 48, 50, 51, 52, 58, 60, 61, 62, 65, 67, 69, 71, 72, 73, 74, 75, 77, 79, 80, 81, 82, 83, 85, 87, 89, 90, 91, 95, 98, 99, 100, 102, 104, 106, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 123, 125, 128, 129, 130, 132, 133, 135, 137, 138, 139, 140, 142, 144, 146, 147, 148, 149, 152, 154, 155, 156, 158, 159, 161, 163, 164, 165, 166, 167, 171, 173, 178, 179
There is no improvement by exchanging points
***** Best design after 3 iterations: 1, 2, 3, 4, 10, 15, 17, 18, 19, 20, 22, 24, 26, 33, 34, 35, 37, 38, 40, 42, 43, 49, 53, 54, 55, 56, 57, 59, 63, 64, 66, 68, 70, 76, 78, 84, 86, 88, 92, 93, 94, 96, 97, 101, 103, 105, 107, 108, 116, 121, 122, 124, 126, 127, 131, 134, 136, 141, 143, 145, 150, 151, 153, 157, 160, 162, 168, 169, 170, 172, 174, 175, 176, 177, 180, 181
with power: 0.999998153329114
>
>


Cerrado region: Quasi−Optimal Design (SSearch, ρ=0.5, Queen)
Latitude
−10
−15
−20
−60
−55
−50
−45
Longitude


Cerrado region: Final Quasi−Optimal Design (SSearch+Exchange, ρ=0.5, Queen
Latitude
−10
−15
−20
−60
−55
−50
−45
Longitude

SIMPLE SEARCH USING NORMAL APPROXIMATION FOR RHO=0.5 AND ROOK RULE

The three isolated nodes are removed at the beginning.

[1] Number of nodes: 178

[1] Complete design power: 0.999919034355779 (0.999913607217003)

[1] * Best 177 -point design when removing node 118 with power 0.9999290712494 (0.99992309332703)
[1] * Best 176 -point design when removing node 6 with power 0.999937803807638 (0.99992931389581)
[1] * Best 175 -point design when removing node 6 with power 0.99994567254828 (0.999937240557082)
[1] * Best 174 -point design when removing node 157 with power 0.999952316697025 (0.99994200819159)
[1] * Best 173 -point design when removing node 127 with power 0.999958030912081 (0.999946502100758)
[1] * Best 172 -point design when removing node 149 with power 0.999962732875532 (0.999950356779081)
[1] * Best 171 -point design when removing node 138 with power 0.999967148394676 (0.99995475753595)
[1] * Best 170 -point design when removing node 157 with power 0.99997092932922 (0.99995786100893)
[1] * Best 169 -point design when removing node 147 with power 0.999975302930192 (0.999962486447082)
[1] * Best 168 -point design when removing node 167 with power 0.999979004953479 (0.999965512396257)
[1] * Best 167 -point design when removing node 166 with power 0.999982209600115 (0.99996880077724)
[1] * Best 166 -point design when removing node 155 with power 0.999985608654206 (0.999972722030138)
[1] * Best 165 -point design when removing node 17 with power 0.999987430069298 (0.99997451441064)
[1] * Best 164 -point design when removing node 10 with power 0.99998978665016 (0.999977237361143)
[1] * Best 163 -point design when removing node 4 with power 0.999991268625934 (0.999979101472302)
[1] * Best 162 -point design when removing node 16 with power 0.999992670356717 (0.999980759830912)
[1] * Best 161 -point design when removing node 10 with power 0.99999407350704 (0.999983015903085)
[1] * Best 160 -point design when removing node 28 with power 0.999994918511467 (0.999984205035424)
[1] * Best 159 -point design when removing node 20 with power 0.999995999769253 (0.999986104794175)
[1] * Best 158 -point design when removing node 34 with power 0.99999677065163 (0.99998740971057)
[1] * Best 157 -point design when removing node 28 with power 0.999997319461585 (0.99998839995429)
[1] * Best 156 -point design when removing node 21 with power 0.999997868561064 (0.99998970466245)
[1] * Best 155 -point design when removing node 28 with power 0.999998256579606 (0.999990510536137)
[1] * Best 154 -point design when removing node 22 with power 0.999998618219482 (0.999991434979395)
[1] * Best 153 -point design when removing node 35 with power 0.999998950570835 (0.999992515167376)
[1] * Best 152 -point design when removing node 122 with power 0.99999912531143 (0.99999301924832)
[1] * Best 151 -point design when removing node 112 with power 0.999999306004297 (0.99999364029855)
[1] * Best 150 -point design when removing node 112 with power 0.999999439927772 (0.99999415644423)
[1] * Best 149 -point design when removing node 16 with power 0.99999953929005 (0.999994639341087)
[1] * Best 148 -point design when removing node 43 with power 0.99999962027568 (0.999994953509906)
[1] * Best 147 -point design when removing node 28 with power 0.999999684817492 (0.999995287243373)
[1] * Best 146 -point design when removing node 52 with power 0.99999974223669 (0.99999557578605)
[1] * Best 145 -point design when removing node 39 with power 0.99999979280327 (0.999995896577693)
[1] * Best 144 -point design when removing node 51 with power 0.999999846085255 (0.999996346809746)
[1] * Best 143 -point design when removing node 30 with power 0.999999878516773 (0.99999661146712)
[1] * Best 142 -point design when removing node 31 with power 0.999999910188756 (0.999996940161624)
[1] * Best 141 -point design when removing node 53 with power 0.999999935627452 (0.999997290938731)
[1] * Best 140 -point design when removing node 51 with power 0.999999950531234 (0.999997525024344)
[1] * Best 139 -point design when removing node 58 with power 0.99999996113521 (0.999997680825143)
[1] * Best 138 -point design when removing node 60 with power 0.999999969994031 (0.999997847164077)
[1] * Best 137 -point design when removing node 73 with power 0.999999978064926 (0.999998029581142)
[1] * Best 136 -point design when removing node 72 with power 0.999999984794568 (0.999998248454899)
[1] * Best 135 -point design when removing node 75 with power 0.9999999883598 (0.99999835096134)
[1] * Best 134 -point design when removing node 84 with power 0.999999991222187 (0.999998486955526)
[1] * Best 133 -point design when removing node 73 with power 0.999999993720458 (0.999998631235603)
[1] * Best 132 -point design when removing node 92 with power 0.999999995285831 (0.999998712887006)
[1] * Best 131 -point design when removing node 101 with power 0.999999996488168 (0.999998798786283)
[1] * Best 130 -point design when removing node 82 with power 0.999999997676396 (0.999998917475222)
[1] * Best 129 -point design when removing node 80 with power 0.999999998516577 (0.999999040660145)

[1] * Best 128 -point design when removing node 78 with power 0.999999998953634 (0.999999111312854)
[1] * Best 127 -point design when removing node 95 with power 0.99999999927109 (0.999999172412068)
[1] * Best 126 -point design when removing node 104 with power 0.999999999504003 (0.999999237051974)
[1] * Best 125 -point design when removing node 102 with power 0.999999999703427 (0.999999327113777)
[1] * Best 124 -point design when removing node 93 with power 0.99999999980521 (0.999999384076177)
[1] * Best 123 -point design when removing node 109 with power 0.999999999875916 (0.999999439009477)
[1] * Best 122 -point design when removing node 116 with power 0.999999999924553 (0.999999502682763)
[1] * Best 121 -point design when removing node 109 with power 0.99999999995116 (0.999999536857992)
[1] * Best 120 -point design when removing node 92 with power 0.99999999996855 (0.999999570167804)
[1] * Best 119 -point design when removing node 69 with power 0.999999999979828 (0.999999599205798)
[1] * Best 118 -point design when removing node 76 with power 0.999999999988415 (0.999999638397385)
[1] * Best 117 -point design when removing node 57 with power 0.999999999994856 (0.999999693549722)
[1] * Best 116 -point design when removing node 46 with power 0.999999999997739 (0.99999973887729)
[1] * Best 115 -point design when removing node 81 with power 0.999999999998818 (0.999999765291063)
[1] * Best 114 -point design when removing node 53 with power 0.999999999999303 (0.999999780583494)
[1] * Best 113 -point design when removing node 63 with power 0.99999999999963 (0.999999800339328)
[1] * Best 112 -point design when removing node 51 with power 0.999999999999838 (0.99999982705084)
[1] * Best 111 -point design when removing node 5 with power 0.999999999999906 (0.999999834231982)
[1] * Best 110 -point design when removing node 11 with power 0.999999999999971 (0.999999863274502)
[1] * Best 109 -point design when removing node 39 with power 0.999999999999983 (0.999999871824194)
[1] * Best 108 -point design when removing node 41 with power 0.99999999999999 (0.999999877980955)
[1] * Best 107 -point design when removing node 76 with power 0.999999999999994 (0.999999883521243)
[1] * Best 106 -point design when removing node 64 with power 0.999999999999997 (0.99999988852305)
[1] * Best 105 -point design when removing node 31 with power 0.999999999999998 (0.999999884566408)
[1] * Best 104 -point design when removing node 51 with power 0.999999999999999 (0.999999887464412)
[1] * Best 103 -point design when removing node 14 with power 1 (0.999999884930395)
[1] * Best 102 -point design when removing node 37 with power 1 (0.99999988262395)
[1] * Best 101 -point design when removing node 64 with power 1 (0.999999880195868)
[1] * Best 100 -point design when removing node 43 with power 1 (0.999999871867858)
[1] * Best 99 -point design when removing node 94 with power 1 (0.99999986914162)
[1] * Best 98 -point design when removing node 21 with power 1 (0.999999858434793)
[1] * Best 97 -point design when removing node 33 with power 1 (0.99999984327782)
[1] * Best 96 -point design when removing node 32 with power 1 (0.999999871266221)
[1] * Best 95 -point design when removing node 29 with power 1 (0.99999985254605)
[1] * Best 94 -point design when removing node 34 with power 1 (0.999999842424194)
[1] * Best 93 -point design when removing node 40 with power 1 (0.999999824737786)
[1] * Best 92 -point design when removing node 39 with power 1 (0.999999821244363)
[1] * Best 91 -point design when removing node 43 with power 1 (0.999999801175616)
[1] * Best 90 -point design when removing node 42 with power 1 (0.999999797288666)
[1] * Best 89 -point design when removing node 52 with power 1 (0.99999977437055)
[1] * Best 88 -point design when removing node 58 with power 1 (0.999999770358103)
[1] * Best 87 -point design when removing node 60 with power 1 (0.999999744552963)
[1] * Best 86 -point design when removing node 61 with power 1 (0.999999707696173)
[1] * Best 85 -point design when removing node 61 with power 1 (0.999999688037994)
[1] * Best 84 -point design when removing node 63 with power 1 (0.999999682382816)
[1] * Best 83 -point design when removing node 68 with power 1 (0.999999646959054)
[1] * Best 82 -point design when removing node 67 with power 1 (0.99999964068565)
[1] * Best 81 -point design when removing node 70 with power 1 (0.999999600745066)
[1] * Best 80 -point design when removing node 69 with power 1 (0.999999594026431)
[1] * Best 79 -point design when removing node 1 with power 0 (0)
[1] * Best 78 -point design when removing node 1 with power 1 (0.999999382838455)
[1] * Best 77 -point design when removing node 1 with power 0 (0)
[1] * Best 76 -point design when removing node 4 with power 1 (0.999999061967004)
[1] * Best 75 -point design when removing node 1 with power 0 (0)
[1] * Best 74 -point design when removing node 3 with power 1 (0.999998576620937)
[1] * Best 73 -point design when removing node 1 with power 0 (0)
[1] * Best 72 -point design when removing node 1 with power 1 (0.99999784419129)

[1] * Best 71 -point design when removing node 1 with power 0 (0)
[1] * Best 70 -point design when removing node 1 with power 0.999999999999998 (0.999996735052874)
[1] * Best 69 -point design when removing node 1 with power 0 (0)
[1] * Best 68 -point design when removing node 4 with power 0.999999999999992 (0.999995062096274)
[1] * Best 67 -point design when removing node 1 with power 0 (0)
[1] * Best 66 -point design when removing node 3 with power 0.999999999999965 (0.999992539689822)
[1] * Best 65 -point design when removing node 1 with power 0 (0)
[1] * Best 64 -point design when removing node 2 with power 0.999999999999845 (0.999988749729435)
[1] * Best 63 -point design when removing node 1 with power 0 (0)
[1] * Best 62 -point design when removing node 1 with power 0.999999999999322 (0.999983052730339)
[1] * Best 61 -point design when removing node 1 with power 0 (0)
[1] * Best 60 -point design when removing node 1 with power 0.999999999997092 (0.999974500050425)
[1] * Best 59 -point design when removing node 1 with power 0 (0)
[1] * Best 58 -point design when removing node 5 with power 0.999999999987756 (0.999961699864542)
[1] * Best 57 -point design when removing node 1 with power 0 (0)
[1] * Best 56 -point design when removing node 1 with power 0.999999999949456 (0.99994259805925)
[1] * Best 55 -point design when removing node 1 with power 0 (0)
[1] * Best 54 -point design when removing node 1 with power 0.99999999979565 (0.999913988982287)
[1] * Best 53 -point design when removing node 1 with power 0 (0)
[1] * Best 52 -point design when removing node 1 with power 0.999999999191798 (0.99987138912728)
[1] * Best 51 -point design when removing node 1 with power 0 (0)
[1] * Best 50 -point design when removing node 1 with power 0.999999996877107 (0.99980805603154)
[1] * Best 49 -point design when removing node 1 with power 0 (0)
[1] * Best 48 -point design when removing node 1 with power 0.999999988226885 (0.999714081107554)
[1] * Best 47 -point design when removing node 1 with power 0 (0)
[1] * Best 46 -point design when removing node 5 with power 0.999999956761405 (0.99957475820725)
[1] * Best 45 -point design when removing node 1 with power 0 (0)
[1] * Best 44 -point design when removing node 1 with power 0.999999845551101 (0.999369034953871)
[1] * Best 43 -point design when removing node 1 with power 0 (0)
[1] * Best 42 -point design when removing node 1 with power 0.999999464389142 (0.999065634920424)
[1] * Best 41 -point design when removing node 1 with power 0 (0)
[1] * Best 40 -point design when removing node 1 with power 0.999998200274111 (0.99861982169275)
[1] * Best 39 -point design when removing node 1 with power 0 (0)
[1] * Best 38 -point design when removing node 8 with power 0.999994153303782 (0.997965541980496)
[1] * Best 37 -point design when removing node 1 with power 0 (0)
[1] * Best 36 -point design when removing node 1 with power 0.999981680228484 (0.997011031089992)
[1] * Best 35 -point design when removing node 1 with power 0 (0)
[1] * Best 34 -point design when removing node 1 with power 0.999944781379072 (0.995618359634806)
[1] * Best 33 -point design when removing node 1 with power 0 (0)
[1] * Best 32 -point design when removing node 1 with power 0.999840361166405 (0.993595632427806)
[1] * Best 31 -point design when removing node 1 with power 0 (0)
[1] * Best 30 -point design when removing node 3 with power 0.999558754873977 (0.99066851555673)
[1] * Best 29 -point design when removing node 1 with power 0 (0)
[1] * Best 28 -point design when removing node 4 with power 0.998838097555022 (0.986451195393454)
[1] * Best 27 -point design when removing node 1 with power 0 (0)
[1] * Best 26 -point design when removing node 3 with power 0.997096514050937 (0.980406885665569)
[1] * Best 25 -point design when removing node 1 with power 0 (0)
[1] * Best 24 -point design when removing node 1 with power 0.993143641543706 (0.97177113645725)
[1] * Best 23 -point design when removing node 1 with power 0 (0)
[1] * Best 22 -point design when removing node 3 with power 0.984769377315273 (0.959525325322036)
[1] * Best 21 -point design when removing node 1 with power 0 (0)
[1] * Best 20 -point design when removing node 1 with power 0.968325101276899 (0.942274191347537)
[1] * Best 19 -point design when removing node 1 with power 0 (0)
[1] * Best 18 -point design when removing node 1 with power 0.93862740061302 (0.91811792590605)
[1] * Best 17 -point design when removing node 1 with power 0 (0)
[1] * Best 16 -point design when removing node 1 with power 0.889725829329216 (0.88460338237502)
[1] * Best 15 -point design when removing node 1 with power 0 (0)

```
[1] * Best 14 -point design when removing node 4 with power 0.816984110479051 ( 0.838540181898833 )
[1] * Best 13 -point design when removing node 1 with power 0 ( 0 )
[1] * Best 12 -point design when removing node 3 with power 0.720159958022197 ( 0.776168077764942 )
[1] * Best 11 -point design when removing node 1 with power 0 ( 0 )
[1] * Best 10 -point design when removing node 1 with power 0.605831373627784 ( 0.692954793822438 )
[1] * Best 9 -point design when removing node 1 with power 0 ( 0 )
[1] * Best 8 -point design when removing node 1 with power 0.486724924277457 ( 0.58448953056749 )
[1] * Best 7 -point design when removing node 1 with power 0 ( 0 )
[1] * Best 6 -point design when removing node 1 with power 0.376660138798075 ( 0.448161307881256 )
[1] * Best 5 -point design when removing node 1 with power 0 ( 0 )
[1] * Best 4 -point design when removing node 1 with power 0.281463968290586 ( 0.287964783357575 )
[1] * Best 3 -point design when removing node 1 with power 0 ( 0 )
[1] * Best 2 -point design when removing node 1 with power 0 ( 0 )

> powers
0.9999291 0.9999378 0.9999457 0.9999523 0.9999580 0.9999627 0.9999671 0.9999709 0.9999753 0.9999790
0.9999822 0.9999856 0.9999874 0.9999898 0.9999913 0.9999927 0.9999941 0.9999949 0.9999960 0.9999968
0.9999973 0.9999979 0.9999983 0.9999986 0.9999990 0.9999991 0.9999993 0.99999940.9999995 0.9999996
0.9999997 0.9999997 0.9999998 0.9999998 0.9999999 0.9999999 0.9999999 1.0000000 1.0000000 1.0000000
1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000
1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000
1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000
1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000
1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000
1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 0.0000000 1.0000000
0.0000000 1.0000000 0.0000000 1.0000000 0.0000000 1.0000000 0.0000000 1.0000000 0.0000000 1.0000000
0.0000000 1.0000000 0.0000000 1.0000000 0.0000000 1.0000000 0.0000000 1.0000000 0.0000000 1.0000000
0.0000000 1.0000000 0.0000000 1.0000000 0.0000000 1.0000000 0.0000000 1.0000000 0.0000000 1.0000000
0.0000000 1.0000000 0.0000000 0.9999998 0.0000000 0.9999995 0.0000000 0.9999982 0.0000000 0.9999942
0.0000000 0.9999817 0.0000000 0.9999448 0.0000000 0.9998404 0.0000000 0.9995588 0.0000000 0.9988381
0.0000000 0.9970965 0.0000000 0.9931436 0.0000000 0.9847694 0.0000000 0.9683251 0.0000000 0.9386274
0.0000000 0.8897258 0.0000000 0.8169841 0.0000000 0.7201600 0.0000000 0.6058314 0.0000000 0.4867249
0.0000000 0.3766601 0.0000000 0.2814640 0.0000000 0.0000000 0.9999190
> max(powers) 1
> which.max(powers) 82

> full.removed.untreated <- c( c(177,33,26), removed.untreated ) #adding the 3 nodes previously removed
> num <- 181
> removed <- full.removed.untreated
> #num <- 8
> #removed <- removed.untreated
> for ( i in 2:(num-2) ) {
+   aux <- sort ( removed[ 1:(i-1) ] )
+   point <- removed[i]
+   for ( j in 1:(i-1) ) if ( aux[j]<= point )  point <- point+1
+   removed [i] <- point
+   }
```

THE FOLLOWING LIST CONTAINS THE REMOVED NODES, WITH THEIR ACTUAL NUMBER (THE ONE THEY HAVE INITIALLY)

```
> removed
  [1] 177  33  26 120   6   7 162 132 155 144 166 154 179 178 165  19  12   4  21  14  37  28  45  39  30  41  32
51 141 130 131  23  61  44  72  58  73  48  50  78  76  85  88 102 101 106 116 104 127 140 115 113 111 137
150 148 135 160 171 161 134  98 110  84  69 123  81  95  79   8  17  64  67 126 107  54  89  24  65 117  77 172
38  60  59  55  68  83  82  92  91 112 125 133 138 139 145 153 152 159 158   1   2   3  11   5  13   9  10  15  16
18  27  20  29  22  31  25  34  35  36  40  49  42  43  46  47  52  53  56  57  62  63  66  80  70  71  74  75  86  87
90 105  93  94  96  97  99 100 103 114 108 121 109 122 118 119 124 136 128 129 142 143 146 147 149 163
151 164 156 157 167 168 169 170 173 174 175 176
```

SIMPLE SEARCH USING NORMAL APPROXIMATION FOR RHO=0.921 AND ROOK RULE

The three isolated nodes are removed at the beginning.

[1] Number of nodes: 178

[1] Complete design power: 1 (NaN)
Warning messages:
1: In lm.morantest.exact(lmobj, area.w, save.M = TRUE, save.U = TRUE) :
  number of zero eigenvalues greater than number of variables
2: In qnorm(p, mean, sd, lower.tail, log.p) : Se han producido NaNs
3: In exactMoran(I, gamma, alternative = alternative, type = type) :
  Out-of-range p-value: reconsider test arguments

[1] * Best 177 -point design when removing node 1 with power 1 (NaN)
[1] * Best 176 -point design when removing node 1 with power 1 (NaN)
[1] * Best 175 -point design when removing node 1 with power 1 (NaN)
[1] * Best 174 -point design when removing node 1 with power 1 (NaN)
[1] * Best 173 -point design when removing node 1 with power 1 (NaN)
[1] * Best 172 -point design when removing node 1 with power 1 (NaN)
[1] * Best 171 -point design when removing node 1 with power 1 (NaN)
[1] * Best 170 -point design when removing node 1 with power 1 (NaN)
[1] * Best 169 -point design when removing node 2 with power 1 (NaN)
[1] * Best 168 -point design when removing node 1 with power 1 (NaN)
[1] * Best 167 -point design when removing node 1 with power 1 (NaN)

SIMPLE SEARCH USING NORMAL APPROXIMATION FOR RHO=1-0.921 AND ROOK RULE

The three isolated nodes are removed at the beginning.

[1] Number of nodes: 178

[1] Complete design power: 0.210498931173051 (0.210447930635253)
[1] * Best 177 -point design when removing node 118 with power 0.211671403572323 (0.211613309927972)
[1] * Best 176 -point design when removing node 129 with power 0.212720486473356 (0.212625856835998)
[1] * Best 175 -point design when removing node 158 with power 0.213738568497623 (0.213618831958856)
[1] * Best 174 -point design when removing node 7 with power 0.214748882853407 (0.214607473138452)
[1] * Best 173 -point design when removing node 6 with power 0.215853879151598 (0.215722100793490)
[1] * Best 172 -point design when removing node 14 with power 0.216802855926687 (0.216592087026629)
[1] * Best 171 -point design when removing node 148 with power 0.217622402998486 (0.217551500153864)
[1] * Best 170 -point design when removing node 137 with power 0.218555750429579 (0.218458023017819)
[1] * Best 169 -point design when removing node 11 with power 0.219343811219346 (0.219354050389925)
[1] * Best 168 -point design when removing node 18 with power 0.220380717714166 (0.220276274905699)
[1] * Best 167 -point design when removing node 15 with power 0.221220575623992 (0.221284757544300)
[1] * Best 166 -point design when removing node 10 with power 0.22220299408713 (0.222240432376059)
[1] * Best 165 -point design when removing node 21 with power 0.223111987149269 (0.223133521017487)
[1] * Best 164 -point design when removing node 21 with power 0.224181035612796 (0.224215500068207)
[1] * Best 163 -point design when removing node 22 with power 0.225155930015702 (0.225267777899719)
[1] * Best 162 -point design when removing node 26 with power 0.225888455931508 (0.226049227382391)
[1] * Best 161 -point design when removing node 52 with power 0.226601962934542 (0.226572085315243)
[1] * Best 160 -point design when removing node 66 with power 0.227736910530433 (0.227696855037192)
[1] * Best 159 -point design when removing node 53 with power 0.228918927691595 (0.229040266388941)
[1] * Best 158 -point design when removing node 66 with power 0.229964446269756 (0.230087418054437)
[1] * Best 157 -point design when removing node 54 with power 0.231055615209399 (0.231234682820041)
[1] * Best 156 -point design when removing node 80 with power 0.232164071903075 (0.232398041009404)
[1] * Best 155 -point design when removing node 121 with power 0.232824132853061 (0.232934756958517)
[1] * Best 154 -point design when removing node 141 with power 0.233548014888439 (0.233703855174704)
[1] * Best 153 -point design when removing node 152 with power 0.234531426177998 (0.234834187962047)
[1] * Best 152 -point design when removing node 151 with power 0.235294927758645 (0.235602389029234)
[1] * Best 151 -point design when removing node 140 with power 0.236576044700017 (0.236941593144635)
[1] * Best 150 -point design when removing node 49 with power 0.237222201928866 (0.237592697197725)
[1] * Best 149 -point design when removing node 35 with power 0.237869558486239 (0.238309322660426)
[1] * Best 148 -point design when removing node 59 with power 0.238545918440865 (0.239149625291937)
[1] * Best 147 -point design when removing node 45 with power 0.239176363137416 (0.239799534490203)
[1] * Best 146 -point design when removing node 35 with power 0.239974203793116 (0.240587169457242)
[1] * Best 145 -point design when removing node 56 with power 0.240943512044213 (0.241655282634414)
[1] * Best 144 -point design when removing node 25 with power 0.241582390331602 (0.242331731997915)
[1] * Best 143 -point design when removing node 31 with power 0.242832649873963 (0.243680612966986)
[1] * Best 142 -point design when removing node 41 with power 0.243849158177096 (0.244651223466892)
[1] * Best 141 -point design when removing node 39 with power 0.244619556242914 (0.245496819457540)
[1] * Best 140 -point design when removing node 61 with power 0.245231381671153 (0.246136072446547)
[1] * Best 139 -point design when removing node 75 with power 0.245831574816804 (0.246893910810256)
[1] * Best 138 -point design when removing node 79 with power 0.246421156059054 (0.247498763790787)
[1] * Best 137 -point design when removing node 123 with power 0.246981707538125 (0.248146099791905)
[1] * Best 136 -point design when removing node 130 with power 0.248189129052336 (0.249339518114983)
[1] * Best 135 -point design when removing node 111 with power 0.249326917492243 (0.250547011256263)
[1] * Best 134 -point design when removing node 102 with power 0.249920177910797 (0.251107603645191)
[1] * Best 133 -point design when removing node 90 with power 0.250550598799404 (0.251793304098111)
[1] * Best 132 -point design when removing node 70 with power 0.251152167494552 (0.252478343438586)
[1] * Best 131 -point design when removing node 90 with power 0.251927115433793 (0.253318584607222)
[1] * Best 130 -point design when removing node 79 with power 0.252839445931490 (0.254224389717900)
[1] * Best 129 -point design when removing node 58 with power 0.253464749350501 (0.25491606892335)
[1] * Best 128 -point design when removing node 47 with power 0.254348208627831 (0.255753802835279)

[1] * Best 127 -point design when removing node 70 with power 0.255025534834428 (0.256545780750057)
[1] * Best 126 -point design when removing node 57 with power 0.255803199360485 (0.257391023158426)
[1] * Best 125 -point design when removing node 75 with power 0.256747195188164 (0.258494498597587)
[1] * Best 124 -point design when removing node 94 with power 0.257631038618478 (0.259421771449078)
[1] * Best 123 -point design when removing node 105 with power 0.258112439422411 (0.259837598086496)
[1] * Best 122 -point design when removing node 95 with power 0.258968015503559 (0.260819246741482)
[1] * Best 121 -point design when removing node 78 with power 0.259431400224768 (0.261384102684559)
[1] * Best 120 -point design when removing node 71 with power 0.260261236984270 (0.262175497045340)
[1] * Best 119 -point design when removing node 63 with power 0.260683231442913 (0.262735669164799)
[1] * Best 118 -point design when removing node 83 with power 0.260989028293080 (0.263118185203614)
[1] * Best 117 -point design when removing node 84 with power 0.261071525852312 (0.263289123325107)
[1] * Best 116 -point design when removing node 75 with power 0.261771791821704 (0.264088678354831)
[1] * Best 115 -point design when removing node 73 with power 0.262466632334949 (0.264943943843599)
[1] * Best 114 -point design when removing node 87 with power 0.263093995482812 (0.265587183256009)
[1] * Best 113 -point design when removing node 93 with power 0.263352621974979 (0.265947300868685)
[1] * Best 112 -point design when removing node 33 with power 0.263175164502745 (0.265851151262065)
[1] * Best 111 -point design when removing node 60 with power 0.262579052670094 (0.265358973969690)
[1] * Best 110 -point design when removing node 16 with power 0.261981243528851 (0.264817092550292)
[1] * Best 109 -point design when removing node 92 with power 0.261381703062542 (0.264306465910059)
[1] * Best 108 -point design when removing node 51 with power 0.260780396111763 (0.263762835280395)
[1] * Best 107 -point design when removing node 16 with power 0.260177286324424 (0.263184070032221)
[1] * Best 106 -point design when removing node 18 with power 0.259572336103256 (0.262628918123887)
[1] * Best 105 -point design when removing node 31 with power 0.258965506550811 (0.262098130539012)
[1] * Best 104 -point design when removing node 94 with power 0.258356757410888 (0.261575277841495)
[1] * Best 103 -point design when removing node 21 with power 0.257746047007366 (0.261099928361603)
[1] * Best 102 -point design when removing node 83 with power 0.256829969620984 (0.260204030483026)
[1] * Best 101 -point design when removing node 78 with power 0.256215445929528 (0.259655632430499)
[1] * Best 100 -point design when removing node 6 with power 0.255295772070878 (0.258816275598693)
[1] * Best 99 -point design when removing node 11 with power 0.25467728975092 (0.258256901396988)
[1] * Best 98 -point design when removing node 95 with power 0.253753907604480 (0.257503873344764)
[1] * Best 97 -point design when removing node 87 with power 0.253131311376407 (0.256913469572783)
[1] * Best 96 -point design when removing node 45 with power 0.252136924308726 (0.256038508952249)
[1] * Best 95 -point design when removing node 38 with power 0.251207985602288 (0.255184278428010)
[1] * Best 94 -point design when removing node 53 with power 0.250579311526526 (0.254606489613187)
[1] * Best 93 -point design when removing node 8 with power 0.249547761836421 (0.253635660694336)
[1] * Best 92 -point design when removing node 3 with power 0.248914473940144 (0.253060386252913)
[1] * Best 91 -point design when removing node 68 with power 0.247878858081689 (0.252014471758262)
[1] * Best 90 -point design when removing node 72 with power 0.247240736571473 (0.251457880032965)
[1] * Best 89 -point design when removing node 60 with power 0.24620086075511 (0.250356479374478)
[1] * Best 88 -point design when removing node 70 with power 0.245557667945554 (0.249910581688539)
[1] * Best 87 -point design when removing node 77 with power 0.244513321415564 (0.248801210391169)
[1] * Best 86 -point design when removing node 80 with power 0.243864799826441 (0.248250946080379)
[1] * Best 85 -point design when removing node 45 with power 0.242777169953754 (0.247186859055264)
[1] * Best 84 -point design when removing node 40 with power 0.241725901844686 (0.246126685071218)
[1] * Best 83 -point design when removing node 45 with power 0.241069205116491 (0.245645442205031)
[1] * Best 82 -point design when removing node 65 with power 0.239974457795220 (0.244526674189884)
[1] * Best 81 -point design when removing node 66 with power 0.238915760544766 (0.243528413895055)
[1] * Best 80 -point design when removing node 53 with power 0.238250263825186 (0.242957640561776)
[1] * Best 79 -point design when removing node 29 with power 0.237097950479897 (0.241804142893988)
[1] * Best 78 -point design when removing node 42 with power 0.235993078433104 (0.2408343033538)
[1] * Best 77 -point design when removing node 35 with power 0.234923781338919 (0.239695611856341)
[1] * Best 76 -point design when removing node 35 with power 0.234245719808378 (0.239145159107396)
[1] * Best 75 -point design when removing node 17 with power 0.233001463953528 (0.237866287958065)
[1] * Best 74 -point design when removing node 17 with power 0.231828230050768 (0.236780407267396)
[1] * Best 73 -point design when removing node 19 with power 0.230587922275067 (0.235503355878686)
[1] * Best 72 -point design when removing node 18 with power 0.231781351671449 (0.236992588420502)

[1] * Best 71 -point design when removing node 1 with power 0 (0)
[1] * Best 70 -point design when removing node 1 with power 0.227675308316435 (0.232772363976575)
[1] * Best 69 -point design when removing node 1 with power 0 (0)
[1] * Best 68 -point design when removing node 1 with power 0.22355335875149 (0.228546340477005)
[1] * Best 67 -point design when removing node 1 with power 0 (0)
[1] * Best 66 -point design when removing node 1 with power 0.219415196318136 (0.22428495318257)
[1] * Best 65 -point design when removing node 1 with power 0 (0)
[1] * Best 64 -point design when removing node 1 with power 0.215260479064002 (0.220046266913438)
[1] * Best 63 -point design when removing node 1 with power 0 (0)
[1] * Best 62 -point design when removing node 4 with power 0.211088825563706 (0.21580163076785)
[1] * Best 61 -point design when removing node 1 with power 0 (0)
[1] * Best 60 -point design when removing node 1 with power 0.206899810149918 (0.211444442381037)
[1] * Best 59 -point design when removing node 1 with power 0 (0)
[1] * Best 58 -point design when removing node 2 with power 0.202692957452034 (0.207125417940223)
[1] * Best 57 -point design when removing node 1 with power 0 (0)
[1] * Best 56 -point design when removing node 2 with power 0.198467736118255 (0.202832138105418)
[1] * Best 55 -point design when removing node 1 with power 0 (0)
[1] * Best 54 -point design when removing node 1 with power 0.194223551570028 (0.198405906815164)
[1] * Best 53 -point design when removing node 1 with power 0 (0)
[1] * Best 52 -point design when removing node 1 with power 0.189959737603033 (0.193985569547604)
[1] * Best 51 -point design when removing node 1 with power 0 (0)
[1] * Best 50 -point design when removing node 6 with power 0.185675546605854 (0.189549539776406)
[1] * Best 49 -point design when removing node 1 with power 0 (0)
[1] * Best 48 -point design when removing node 5 with power 0.181370138111071 (0.185089578002175)
[1] * Best 47 -point design when removing node 1 with power 0 (0)
[1] * Best 46 -point design when removing node 4 with power 0.177042565320946 (0.180566854913059)
[1] * Best 45 -point design when removing node 1 with power 0 (0)
[1] * Best 44 -point design when removing node 3 with power 0.172691759154050 (0.176036485386187)
[1] * Best 43 -point design when removing node 1 with power 0 (0)
[1] * Best 42 -point design when removing node 1 with power 0.168316509233234 (0.171454819531540)
[1] * Best 41 -point design when removing node 1 with power 0 (0)
[1] * Best 40 -point design when removing node 1 with power 0.163915441063485 (0.166853790430495)
[1] * Best 39 -point design when removing node 1 with power 0 (0)
[1] * Best 38 -point design when removing node 4 with power 0.159486988415318 (0.162181128882741)
[1] * Best 37 -point design when removing node 1 with power 0 (0)
[1] * Best 36 -point design when removing node 1 with power 0.155029359603835 (0.157526445785174)
[1] * Best 35 -point design when removing node 1 with power 0 (0)
[1] * Best 34 -point design when removing node 6 with power 0.150540495891502 (0.152772987995672)
[1] * Best 33 -point design when removing node 1 with power 0 (0)
[1] * Best 32 -point design when removing node 1 with power 0.146018019571092 (0.147961757952102)
[1] * Best 31 -point design when removing node 1 with power 0 (0)
[1] * Best 30 -point design when removing node 1 with power 0.141459168282000 (0.143084535757577)
[1] * Best 29 -point design when removing node 1 with power 0 (0)
[1] * Best 28 -point design when removing node 3 with power 0.136860710567889 (0.138143817952927)
[1] * Best 27 -point design when removing node 1 with power 0 (0)
[1] * Best 26 -point design when removing node 3 with power 0.132218835214591 (0.133151785677457)
[1] * Best 25 -point design when removing node 1 with power 0 (0)
[1] * Best 24 -point design when removing node 3 with power 0.127529002793096 (0.128034350909205)
[1] * Best 23 -point design when removing node 1 with power 0 (0)
[1] * Best 22 -point design when removing node 5 with power 0.122785740626872 (0.122847137724949)
[1] * Best 21 -point design when removing node 1 with power 0 (0)
[1] * Best 20 -point design when removing node 4 with power 0.117982349044905 (0.117573391912691)
[1] * Best 19 -point design when removing node 1 with power 0 (0)
[1] * Best 18 -point design when removing node 1 with power 0.113110460303191 (0.112145708087191)
[1] * Best 17 -point design when removing node 1 with power 0 (0)
[1] * Best 16 -point design when removing node 1 with power 0.108159334866132 (0.106565489181409)
[1] * Best 15 -point design when removing node 1 with power 0 (0)

```
[1] * Best 14 -point design when removing node 6 with power 0.103114646861072 ( 0.100768923222120 )
[1] * Best 13 -point design when removing node 1 with power 0 ( 0 )
[1] * Best 12 -point design when removing node 1 with power 0.0979561637764845 ( 0.0947938532836672 )
[1] * Best 11 -point design when removing node 1 with power 0 ( 0 )
[1] * Best 10 -point design when removing node 1 with power 0.0926526932919639 ( 0.088507834816731 )
[1] * Best 9 -point design when removing node 1 with power 0 ( 0 )
[1] * Best 8 -point design when removing node 3 with power 0.0871490267543846 ( 0.0818360371483855 )
[1] * Best 7 -point design when removing node 1 with power 0 ( 0 )
[1] * Best 6 -point design when removing node 1 with power 0.081323222632521 ( 0.0746658913225509 )
[1] * Best 5 -point design when removing node 1 with power 0 ( 0 )
[1] * Best 4 -point design when removing node 1 with power 0.0747809974784828 ( 0.066803656256986 )
[1] * Best 3 -point design when removing node 1 with power 0 ( 0 )
[1] * Best 2 -point design when removing node 1 with power 0 ( 0 )
Hubo 50 o  más avisos (use warnings() para ver los primeros 50)
> powers
   [1]  0.21167140  0.21272049  0.21373857  0.21474888  0.21585388  0.21680286  0.21762240  0.21855575
0.21934381 0.22038072 0.22122058 0.22220299 0.22311199 0.22418104 0.22515593 0.22588846 0.22660196
0.22773691 0.22891893 0.22996445 0.23105562 0.23216407 0.23282413 0.23354801 0.23453143 0.23529493
0.23657604 0.23722220 0.23786956 0.23854592 0.23917636 0.23997420 0.24094351 0.24158239 0.24283265
0.24384916 0.24461956 0.24523138 0.24583157 0.24642116 0.24698171 0.24818913 0.24932692 0.24992018
0.25055060 0.25115217 0.25192712 0.25283945 0.25346475 0.25434821 0.25502553 0.25580320 0.25674720
0.25763104 0.25811244 0.25896802 0.25943140 0.26026124 0.26068323 0.26098903 0.26107153 0.26177179
0.26246663 0.26309400 0.26335262 0.26317516 0.26257905 0.26198124 0.26138170 0.26078040 0.26017729
0.25957234 0.25896551 0.25835676 0.25774605 0.25682997 0.25621545 0.25529577 0.25467729 0.25375391
0.25313131 0.25213692 0.25120799 0.25057931 0.24954776 0.24891447 0.24787886 0.24724074 0.24620086
0.24555767 0.24451332 0.24386480 0.24277717 0.24172590 0.24106921 0.23997446 0.23891576 0.23825026
0.23709795 0.23599308 0.23492378 0.23424572 0.23300146 0.23182823 0.23058792 0.23178135 0.00000000
0.22767531 0.00000000 0.22355336 0.00000000 0.21941520 0.00000000 0.21526048 0.00000000 0.21108883
0.00000000 0.20689981 0.00000000 0.20269296 0.00000000 0.19846774 0.00000000 0.19422355 0.00000000
0.18995974 0.00000000 0.18567555 0.00000000 0.18137014 0.00000000 0.17704257 0.00000000 0.17269176
0.00000000 0.16831651 0.00000000 0.16391544 0.00000000 0.15948699 0.00000000 0.15502936 0.00000000
0.15054050 0.00000000 0.14601802 0.00000000 0.14145917 0.00000000 0.13686071 0.00000000 0.13221884
0.00000000 0.12752900 0.00000000 0.12278574 0.00000000 0.11798235 0.00000000 0.11311046 0.00000000
0.10815933 0.00000000 0.10311465 0.00000000 0.09795616 0.00000000 0.09265269 0.00000000 0.08714903
0.00000000 0.08132322 0.00000000 0.07478100 0.00000000 0.00000000 0.21049893
> max(powers)
[1] 0.2633526
> which.max(powers)
[1] 65
```

THE FOLLOWING LIST CONTAINS THE REMOVED NODES, WITH THEIR ACTUAL NUMBER (THE ONE THEY HAVE INITIALLY)

```
> removed
177  33  26 120 132 162   7   6  16 155 144  13  22  19  12  29  30  32  38  65  80  67  82  69  98 142 166 179
178 165  62  48  77  59  49  76  37  45  58  56  89 105 110 160 171 147 136 123 100 125 112  86  71 103  87
113 138 153 140 117 107  95 128 130 116 114 139 150  50  93  23 154  81  24  28  52 164  36 148 137   8  17
174 161  83  70  99  11   3 133 145 119 143 159 169  90  78  92 135 146 109  57  88  72  73  39  40  43  42   1
2   4   5   9  10  14  15  18  27  20  21  25  34  31  41  35  44  46  47  51  63  53  64  54  66  55  68  60  61  74  75
79  94  84  85  91 106  96  97 101 102 104 115 108 121 111 124 118 131 122 134 126 127 129 141 149 163
151 152 156 157 158 170 167 168 172 173 175 176
```